Correlated Default Risk And Bank Risk-Taking Where Bank Assets Are Risky Debt claims *

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Abstract

According to the basic structural approach bank aggregated asset value follows a log-normal process. As a result shareholders are motivated to transfer wealth from bondholders by engaging in risky projects. However, bank assets usually consist of a portfolio of risky loans to several corporate borrowers. Using these assumptions, we show that risk shifting is limited to states in which the debtor is in financial distress. A diversified loans portfolio decreases risk shifting. Moreover, the optimal level of asset risk decreases as the portfolio is less correlated. Furthermore, we show that for a given leverage ratio, the level of risk of a borrower may increase if the quality of the bank's entire portfolio deteriorates.

Keywords: Risk taking, Asset risk, Financial institutions, Stress test, Leverage

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1 Introduction

The structural approach for pricing corporate liabilities, developed by Merton [1974], views debt and equity as contingent claims on the firm's assets.[1] In this framework, since the value of a firm's stock, which is equivalent to a call option, is positively related to the underlying asset's volatility, a stockholder aligned manager would be motivated to increase asset risk (Jensen and Meckling [1976] and Galai and Masulis [1976]). However, rational creditors will consider these incentives when determining credit conditions. The financial literature suggests that banks are especially good at setting credit conditions and covenants, and monitoring their borrowers to limit risk shifting (Brealey et al. [1977], Campbel and Kracaw [1980], Diamond [1984], and Fama [1985]).

In contrast, empirical and theoretical work suggests that creditors' ability to limit risk shifting is more restricted in financial institutions as creditors are small and dispersed and bondholders may have explicit (deposit insurance) or implicit (too big to fail) guaranties. Moreover, banks are more complex and opaque with greater information asymmetries than non-financial firms; this makes bank asset risk hard to observe and assess while being easy to change and manipulate.[2]

In a recent paper, Nagel and Purnanandam [2015] claimed that, since bank's assets are risky loans, their value is capped.[3] Therefore the value of the bank's stock is not equivalent to a call option on its asset as implied by the basic structural approach. Instead, a bank's assets are contingent claims on the value of their debtors' assets (see Figure 1). Thus, bank equity is economically equivalent to an 'option-on-option' with a payoff function identical to that of subordinated debt, which can be replicated by a 'bull spread' strategy (Black and Cox [1976]).[4] Moreover, since bank assets are risky debt claims with limited upside, asset volatility depends on the debtor's asset value. Consequently, when assuming that bank assets follow a log-normal stochastic process with a constant volatility, the bank's probability of default and its costs of deposit insurance in bad times, when leverage is high, are underestimated.

Relying on the same assumption that bank assets are risky debt claims with limited upside (Nagel and Purnanandam [2015] and Dermine and Lajeri [2001]) and using the option pricing approach (Black and Scholes [1973] and Merton [1974]) we analyze the risk taking motivation of a bank's stockholder using a principal-agent model. We assume that the debtor - that is, the borrowing corporation - can shift the risk

[1] A risky corporate bond is economically equivalent to a long position in a risk-free bond and a short position in a European put option on the firm's assets, with a strike price equal to the face value of its debt. Similarly, a firm's stock is economically equivalent to a call option on the value of the firm's assets with a strike price equal to the face value of its debt.

[2] Caprio and Levine [2002] discussed the corporate governance of banks and their opaqueness. Morgan [2000] found that bond analysts disagree more over bonds issued by banks than by non-financial firms suggesting that banks tend to be more opaque than non-financial firms.

[3] A similar analysis was suggested earlier by Dermine and Lajeri [2001].

[4] This strategy consists of a long position in a call option on the corporation's asset value, with a strike price equal to the face value of the bank's debt and a short position in a call option on the corporation's asset value, with a strike price equal to the face value of the debtor's debt.

of its assets from its initial level, which can only be done with the consent of the creditor - the bank. The corporation and the bank are both managed by an equityholder aligned manager, whose goal is to maximize equity value. We determine if the stockholders are motivated to increase asset risk from its initial level and characterize the equilibrium level of asset risk. Moreover, we analyze the effect of risk shifting on the liabilities of the bank and the corporation, the cost of deposit insurance and the bank's probability of default.

We prove that, in contrast to the basic agent theory in which the stockholder is motivated to shift risk in both solvent and insolvent states (Jensen and Meckling [1976]), in our setting, the bank's stockholder is motivated to shift risk only when the debtor is in financial distress, that is, when the value of the debtor's assets is below the face value of the debtor's debt. Specifically, using a closed-form solution, we prove that risk shifting may occur only if the value of the debtor's assets is lower than the geometric mean of the face values of the bank's and debtor's debt.

As the value of the debtor's equity increases with asset risk, its stockholder would be motivated to increase asset risk in any state (Jensen and Meckling [1976]). In contrast, the creditor's risk taking motivation depends on the value of the debtor's assets. Here we distinguish between two cases. In the first case, where the value of the debtor's assets is above the risk shifting threshold, located between the face values of the debtor's debt and the creditor's debt, an increase in asset risk will decrease the value of the bank's equity (Figure 5). An equityholder aligned bank manager is motivated to monitor the debtor tightly and restrict risk shifting by the debtor's stockholder. In equilibrium, risk shifting would not occur because, one of the stockholders - either of the debtor or of the creditor- would lose if the initial level of risk is changed.

In the second case, where the debtor is in financial distress and the value of the debtor's assets is below the threshold described above, the relationship between asset risk and the value of the bank's stock is hump-shaped (Figure 6). In this case, the bank's stockholder would willingly tolerate a shift in asset risk by the debtor, the corporation, if the shift increases its equity value. Therefore, in equilibrium the chosen level of asset risk is the level that maximizes the bank stockholder's position, that is the maximum level of the humped curve.

In our setting, the level of asset risk-set in equilibrium depends on the bank's initial capital ratio. With all else equal, as the bank's equity layer increases, the stockholder's motivation to shift risk decreases. The result strengthens the argument for a higher capital adequacy in banks (Admati et al. [2013]; Miles et al. [2013] and Turner [2010]), since the increase in a bank's capital not only decreases its probability of default and the cost of deposit insurance through the higher capital buffer, but also reduces the risk taking motivation of the bank's stockholders.[5] Further, we show that an increase in leverage due to a negative shock to the value of debtor's assets, increases the probability of risk shifting. This can be explained intuitively, as the value of the corporation's assets decreases the payoff from an upside movement increases while the

[5] The costs of issuing more equity is beyond the scope of our paper. De Nicolò et al. [2012] argued that even if capital requirements are initially beneficial, there is a point at which further increases become costly, reducing lending, efficiency, and welfare.

downside risk is constant. In most of the literature of the structural approach for pricing corporate liabilities the basic assumption is that bank assets are composed of a single loan. However, in practice bank's assets are composed of a diversified portfolio of individual loan with different asset risk and correlation which is less than one. An exceptional is Flannery [1989] who considers "the properties of individual assets within the portfolio. The analysis highlights the ambiguous impact that increased individual loan risks has on the value of a bank's deposit insurance. Chen et al. [2006] extended and elaborated Flannery [1989] framework by analyzing the cost of deposit insurance for the case of bank's assets which are a portfolio of risky loans. When we study the effect of diversification, we find that a diversified loans portfolio decreases risk shifting. Moreover, the optimal level of asset risk decreases as the portfolio is less correlated. Furthermore, we show that for a given leverage ratio, the level of risk of a borrower may increase if the quality of the bank's entire portfolio deteriorates.

Finally, following the 2007-2009 financial crisis, federal regulators undertook a unique supervisory capital assessment program under which large, complex bank-holding companies are expected to run stress tests to prove that they can manage real economic activity even under adverse economic conditions. By calibrating our model to typical market data, we demonstrate that not accounting for the possibility of risk shifting in bad economic states may lead to severe underestimation of the cost of deposit insurance. However, since risk shifting when bank assets are risky debt claims is limited to states in which the value of the debtor's assets is below the face value of the debtor's debt, the effect of risk shifting on a bank's probability of default is relatively minor.

Literature review The literature regarding financial institutions' asset risk focuses on several major conflicts: the conflict between bank stockholders and depositors, which are usually represented by a benevolent regulator; the conflict between a bank and its debtors and the conflict between a bank's executives and its claimholders (stock and debtholders).

The conflict between equityholders and debtholders regarding the level of asset risk is described in the early work of Jensen and Meckling [1976] and Galai and Masulis [1976]. A firm's equityholders are motivated to increase asset risk-since their payoff is a convex function of firm value due to the limited liability principle. Risk shifting increases equity value at the expense of creditors. Thus, by selecting riskier projects in a way that is not anticipated by creditors, equityholders can transfer wealth to their own benefit. Similarly, we assume that the stockholders of both the borrowing corporation and the lending bank are trying to maximize the value of their holding by choosing the level of asset risk without regard to the total value of the firm's assets. However, in our model bank risk can only be increased by changing the asset risk of the debtor. Therefore, risk shifting from the initial level of asset risk can occur only with the consent of the stockholders of both - the debtor and the creditor. This assumption requires us to account for the strategic interaction between these stockholders.

The conflict between stockholders and bondholders in financial institutions appears to be mainly focused on the ability of the depositors and their representatives, the regulators, to monitor the level of a bank's asset risk (Acharya et al. [2016]; Agoraki et al. [2011]; Hilscher et al. [2015]; Delis and Staikouras [2011]). However, over the last few decades, as the size and complexity of financial firms has increased, the regulators' ability to control and monitor bank's asset risk has deteriorated (Berger et al. [2000]; DeYoung et al. [2001]; Caprio and Levine [2002]; and Morgan [2000]). Moreover, the existence of deposit insurance motivated banks to take more risks and effectively shifted the agent problem from the bank's creditors to the regulators (Demirgüç-Kunt and Detragiache [1998]; Cooper and Ross [2002] and Allen et al. [2015]).

The assumption that a bank's assets are risky debt claims limits the conflict between bank creditors and equityholders to states in which the bank's debtor is in financial distress, or more precisely, where the value of its assets is below the geometric mean of the face values of the debtor's and creditor's debts. This result holds true even though throughout most of our analysis we assume that a bank's debtholders and regulators are unable to monitor and restrict the bank's asset risk.

The conclusion that banks are motivated to shift asset risk in states of financial distress appears several times in the theoretical literature (Bruche and Llobet [2014]; Diamond and Rajan [2011]). The idea that a stockholder's motivation to shift risk increases in states of financial distress is referred to as gambling for resurrection (Dewatripont et al. [1995]; Mishkin [1992]). When the value of a bank's equity is depleted, a bank may willingly take on large risks even if these risks are associated with low expected returns. If these gambles pay off, the bank may survive; if they do not, the bank would have been broke anyway (Boyd and Hakenes [2014]). While the idea of risk shifting in financial distress appears in these papers as well, we consider the interaction that occurs between the stockholders of the bank and the debtor when bank assets are risky debt claims.

In this setting, we ignore possible deviations between the incentives of the bank's manager and claimholders, which can lead to risk shifting in solvent states, as described in a number of recent works, mostly written after the recent financial crisis. These works describe the positive effect of equity-based compensation on risk taking (Bhattacharyya and Purnanandam [2011] and Cheng et al. [2010]) and the negative effect of debt-like compensation, which is sometimes referred to as inside debt (Edmans and Liu [2011], Raviv and Sisli-Ciamarra [2013]).[6]

Finally, this paper is related to the debate that arose following the 2007-2009 financial crisis regarding the appropriate ways to regulate 'too big to fail' financial institutions. First, the paper highlights the relationship between leverage and risk taking, strengthening the argument for a higher bank capital ratio (Admati et al. [2013] and Admati et al. [2011]). Second, after the financial crisis, regulators around the world started conducting periodic stress tests for financial institutions. These are forward-looking assessments designed

[6] In the financial literature there are contradicting evidence regarding the effect of executive compensation on risk taking (Murphy [2013]).

to determine if a bank would have adequate capital to withstand negative shocks in the future (Goldstein and Sapra [2014], Goldstein and Leitner [2015], Peristiani et al. [2010], Bayazitova and Shivdasani [2012]), Gofman [2011] and Greenlaw et al. [2012]). Our model contributes to this approach as we estimate a bank's resilience under the assumption that the bank's assets are risky debt claims.

The rest of the paper is organized as follows. Section 2 presents the liability structures of the corporation and bank and the valuation of the different claimholders' positions. Section 3 presents our equilibrium model for risk shifting. Section 5 includes a quantitative analysis of the effects of capital and asset value on risk shifting as well as the effect of diversified loans portfolio. Section 6 concludes the paper.

2 Liability Structure and Valuation

This section describes the liability structures of both the bank and the corporation, defines the value of the claimholders' positions and defines the bank's probability of default and the cost of deposit insurance. We consider a single corporation and bank financed by equity and debt. Throughout the paper it is assumed that the bank and corporation are both managed by equity-aligned managers aiming to maximize the value of equity. Thus we refrain from an agency problem between the equityholder and manager, and focus on the agency problem between the equityholder and debtholder. It is also assumed that the managers of the bank and of the corporation are both fully informed, thus avoiding problems of information asymmetry.

2.1 The Corporation's Liability Structure

The corporation is funded by an equity with a market value of S^C and a single loan with a face value of F^C and market value of B^C.[7] The loan is a zero-coupon loan maturing at time T and the bank is the sole creditor. The value of the firm's assets, V^C, follows a geometric Brownian motion according to the following equation:

$$dV^C = \mu V^C dt + \sigma V^C dW \tag{1}$$

where μ is the instantaneous expected return on the corporation's assets, σ is the volatility of the corporation's assets, and dW is a standard Wiener process. The event of default occurs at debt maturity, T, if the value of the asset, V_T, is lower than the face value of the debt. If default occurs, the creditor takes over the firm without incurring any distress costs and realizes the residual assets of the firm, V_T^C. Otherwise, the debt is fully paid and the creditor, the bank, receives the entire face value of the debt, F^C. The payoff to the corporation's debtholder can be expressed as $B_T^C = \min(V_T^C, F^C)$, or when rearranged:

$$B_T^C = F^C - \max(F^C - V_T^C, 0). \tag{2}$$

[7] To keep the notation as simple as possible, all variables without subscripts refer to time t.

As developed by Merton [1974], Equation (18) is equivalent to the payoff of a risk-free debt minus a European put option. Therefore, the present value of the corporate debt is given by:

$$B^C = F^C e^{-r(T-t)} - Put(V^C, F^C, \sigma, T-t) \tag{3}$$

where r is the risk-free rate and $Put(V^C, F^C, \sigma, T-t)$ is the price of a European put option according to the equations developed by Black and Scholes [1973].

As the equity is the residual claim, its payoff at debt maturity is:

$$S_T^C = \max\left(V_T^C - F^C, 0\right). \tag{4}$$

The value of the corporation's stock prior to debt maturity can be replicated by a European call option on the value of the corporation's assets, with a strike price equal to its face value of debt (Galai and Masulis [1976]):

$$S^C = Call(V^C, F^C, \sigma, T-t). \tag{5}$$

where $call(V^C, F^C, \sigma, T-t)$ is the price of a European call option as developed by Black and Scholes [1973].

2.2 The Bank's Liability Structure

The bank is funded by equity with a market value of S^B and zero-coupon debt with a face value of F^B and market value of B^B. It is assumed that the debtholders are many small depositors, who are unwilling or unable to monitor the bank manager's actions. Therefore, the bank's stockholder is the only claimholder who controls the level of the debtor's asset risk. As discussed in Marcus and Shaked [1984], due to the periodic frequency of supervisory audits, bank deposits are analogous to a debt claim with an effective maturity equal to the examination interval which we assume to be T.

As the bank has a single asset, the value of the bank's assets is equal to the value of the loan, $V^B = B^C$. Accordingly, the bank's assets payoff at maturity, V_T^B, is identical to the payoff of the corporation's debt, as expressed in Equation 18. Therefore, the value of the bank's assets prior to debt maturity, V^B, can be replicated by a long position in a risk-free debt and a short position in a European put option as described in Equation 19. As the bank is in possession of all information regarding the corporation's assets and actions, the bank monitors the firm effectively. Consequently, the corporation can only shift risk from the initial level of asset risk set in the loan contract with the bank's consent (as explained in detail in Section 3.1).

The payoff at maturity to the bank's debtholder is the minimum between the value of the bank's assets and the face value of the bank's debt, F^B, and can be expressed as:

$$B_T^B = \min\left(V_T^B, F^B\right) = \min\left(F^C - \max\left(F^C - V_T^C, 0\right), F^B\right) \tag{6}$$

It is assumed that the bank is solvent at the time of debt issuance and at regulatory audits. If the bank is insolvent at the time of audit, the regulator will use the bank's residual assets to service the debt. This assumption requires the existence of a positive gap between the face value of the corporate debt, F^C, and the face value of the bank's debt, F^B. The corporate loan, which is the bank's single asset, is financed by both equity and debt. Therefore, the face value of its loan is always lower than the total face value of the corporation. Under this assumption, Equation 6 can be expressed as:

$$B_T^B = F^B - \max(F^B - V_T^C, 0). \tag{7}$$

The payoff of the bank's debt can be replicated by a long position in a risk-free debt with a face value of F^B and a short position in a European put option on the corporation's assets, with a strike price equal to the face value of the bank's debt. Therefore, the value of the bank's debt prior to debt maturity can be expressed as:

$$B^B = F^B e^{-r(T-t)} - Put(V^C, F^B, \sigma, T-t). \tag{8}$$

As the bank's equityholder is the residual claimant, its payoff at maturity is $S_T^B = max[V_T^B - B_T^B, 0]$. If the bank is solvent at maturity, the equityholder receives the payoff $F^C - F^B$, which is the maximum payoff that the bank's equityholder can receive. This differs from the basic structural approach in which the equityholders' payoff is unbounded.

The bank equityholder's payoff at maturity can be rearranged and expressed as:

$$S_T^B = max(V_T^C - F^B, 0) - max(V_T^C - F^C, 0). \tag{9}$$

This payoff can be replicated by a long position in a call option, with a strike price equal to the face value of the bank's debt, F^B, and by a short position in a call option, with a strike price equal to the face value of the corporation's debt, F^C. Therefore, the value of the bank's equity prior to debt maturity is:

$$S^B = Call(V^C, F^B, \sigma, T-t) - Call(V^C, F^C, \sigma, T-t). \tag{10}$$

The above option position is known as a 'bull spread' strategy; it was used to model junior debt by Black and Cox [1976]. In our setting, the bank's equity acts similarly to a junior debt. Since the bank's asset is a single corporate loan and the corporation's single creditor is the bank, the bank's creditor has seniority in the event of the corporation's default. The bank's equityholder receives a positive residual only if the bank's creditor is fully paid. If the creditor is fully paid by the debtor, that is, if the bank's stockholder receives the amount of $F^C - F^B$, the corporation's stockholder receives a positive value - the residual assets of the corporation. Thus, the bank's equity is equivalent to a mezzanine debt in the corporation's capital structure. The position of each of the bank's claimholders and the value of the bank's assets at maturity are described in Figure 1.

2.3 Probability of Default

In our setting the bank is in default if the value of the corporation's assets is below the face value of the bank's debt at debt maturity. As in Merton [1974], the bank's risk-neutral probability of default can be calculated using the following equation:

$$Prob(V^C < F^B) = PD^B = N(-d_2^B). \tag{11}$$

where $d_2^B = \frac{ln\left(\frac{V}{F^B}\right)+(r-\frac{1}{2}\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$. The bank's probability of default decreases with the value of the corporation's assets as described by Figure 2. Equation 11 can be used to calculate the bank's probability of default following a negative shock to the value of the corporation's assets. This is equivalent to conducting a stress test for the bank under adverse scenarios. As discussed in Section 3, when we take into account the possibility of risk shifting by using our equilibrium model, the probability of default may increase further.

2.4 The Cost of Deposit Insurance

In the existence of deposit insurance, if the value of the bank's assets is below the face value of its deposit at maturity, the guarantor compensates the depositors with the difference between the two. The value of the deposit insurance equals the difference between the value of a risk-free debt with a notional amount equal to the face value of the secured deposit, and the value of the bank's risky debt. Thus, as discussed in Merton [1977] and Crouhy and Galai [1991], the insurance is equivalent to a long put option on the corporation's assets with a strike price equal to the face value of the bank's debt: $DI = Put(V^C, F^B, \sigma, T-t)$. The value of the deposit insurance per dollar of insured deposits is defined as:

$$DIPD = \frac{Put(V^C, F^B, \sigma, T-t)}{F^B}. \tag{12}$$

The value of the deposit insurance increases with the corporation's asset risk, σ, and decreases with the corporation's asset value, V^C (Figure 3). As shown in Section 3, when taking into account the increase in asset volatility as a result of a decrease in the value of the debtor's assets, the increase in the value of deposit insurance may be substantial.

3 An Equilibrium Model For Risk Shifting

3.1 The Framework of Analysis

In our model the bank sets the face value of the corporate loan, F^C, to account for the corporation's initial asset risk, $\sigma_{Initial}$. When the loan is initiated, the bank's liabilities, the stock and the secured deposit, are fairly priced according to the corporation's asset risk and leverage.

It is assumed that the stockholders of both the corporation and the bank are trying to maximize the value their holdings without regard to the effect on the value of the bank or corporation assets. Thus, in the event of a risk shifting opportunity, which can increase the value of their holding at the expense of other claimholders, the bank depositor or the corporation debtholder, the stockholders would not reject it.

We assume that, some time after the contract is set, an exogenous shock to the value of the corporation's assets occurs, decreasing or increasing the value of the bank's assets. The change in asset value may change the sensitivity to asset risk of the bank's stock. The bank's stockholder might be willing to tolerate a change in asset risk by the debtor's stockholders. Therefore, the equilibrium solution for the decision variables and the value of the stockholders' positions are determined by a two-step backward induction. First, the corporation's stockholder chooses the level of asset risk, σ^*, which maximizes the value of her position, S^C. This decision is made by considering the upper and lower bounds on asset risk set by the bank's stockholder: $\sigma \in [\sigma_{LBound}, \sigma_{UBound}]$. However, the domain of asset risk must contain the initial level of asset risk, $\sigma_{Initial}$. Thus, the bank cannot force the corporation to change its asset risk from the initial level set in the terms of the contract. A shift in the level of asset risk may occur only if the two counterparts - the stockholders of the corporation and the bank - are both better off.

Second, after analyzing the decision of the corporation's stockholder, the bank's stockholder chooses the upper and lower bounds on asset risk, σ^*_{LBound} and σ^*_{UBound}. In this domain, the value of bank stockholder's position, S^B, is always greater than or equal to its initial value. Both σ^*_{LBound} and σ^*_{UBound} are positive and limited from above by the existing technologies governing the corporation's asset risk. If the stockholders of the bank and the corporation both chose a strategy and cannot both benefit from changing one strategy while the other remains unchanged, then the set of strategy choices and corresponding payoffs constitute a Nash equilibrium. We define the set of parameters and payoffs in such an equilibrium as: $\left\langle \left(\sigma^*, \sigma^*_{LBound}, \sigma^*_{UBound}\right), \left(S^{B*}, S^{C*}\right) \right\rangle$.

3.2 Risk Shifting in Equilibrium

The Risk Preference of the Corporation's Equityholder The value of the corporation's stock, a call option on the value of its assets, increases with asset volatility, as illustrated in Figure 4. This result stems from the fact that the value of a call option increases with asset risk, as shown in Jensen and Meckling

[1976]. Thus, in all states of the world the corporation's stockholder would be motivated to increase asset risk from its initial level. However, as discussed above, such a shift can occur only with the consent of the bank's stockholder. Moreover, a lower bound on asset risk, σ^*_{LBound}, set by the bank below the initial level of asset risk is always unbinding. Therefore, we ignore it throughout the rest of this paper.

Bank Equityholder Risk Preference As demonstrated in Appendix A.1, the sensitivity of the bank's equity value to the debtor's asset risk is divided into two segments defined by the following threshold of the corporation's asset value:

$$V^* \equiv (F^B F^C)^{\frac{1}{2}} e^{-r(T-t)} \tag{13}$$

In the first case, when the value of the corporation's assets is above this threshold, V^*, the bank equityholder's sensitivity to asset risk is negative (Figure 5). Hence, the value of the bank's stock decreases monotonically with asset risk. In this case, the bank stockholder would not tolerate any increase in asset risk by the corporation's stockholder, and the upper bound on asset risk is equal to the initial level of asset risk, $\sigma_{Initial}$. Conversely, the position of the corporation's stockholder increases with asset risk. Therefore in equilibrium her choice would be to set asset risk at its maximum possible level, which is equal to the initial level of asset risk. The result of these conflicting interests is an equilibrium in which asset risk is equal to the initial level of asset risk, $\sigma_{Initial}$.

In the second, complementary case, in which the value of the assets is below the threshold, V^*, the relationship between the value of the bank stock and asset risk is non-monotonic, and has a hump-shaped structure (Figure 6). The value of the bank's equity first increases until it reaches its maximum value, then begins to decrease as asset risk increases further. The bank equityholder's position has a constrained maximum, $\sigma_{max} > 0$, which is defined in Appendix A.1 as:

$$\sigma_{max} \equiv \arg\max_{\sigma} S^B(V^C) = \sqrt{\frac{1}{T-t} \ln\left(\frac{F^B F^C}{(V^C)^2}\right) - 2r} \tag{14}$$

The level of the corporation's asset risk which maximizes the position of the bank's equityholder for different asset values is presented in Figure 7.

Proposition 1. *When the value of the corporation's assets is below the threshold level, V^*, as defined in Equation 13, risk shifting to a higher level than the initial level of asset risk may occur in equilibrium. Thus, the condition is a necessary, but not sufficient condition for risk shifting.*

Proof. When the value of the corporation's assets is below the threshold, V^*, the value of the bank's equity is hump-shaped with respect to asset risk and the bank equityholder's position has a constrained maximum, σ_{max}. If $\sigma_{max} < \sigma_{Initial}$ then the bank's equityholder would be better off by shifting to the lower level of asset risk, σ_{max}. However, the corporation's equityholder would prefer not to reduce asset risk from its initial level because the value of her position increases with asset risk. Therefore, the equilibrium solution in such a case is the initial level of asset risk, $\sigma_{Initial}$, and risk shifting would not occur. Conversely, if $\sigma_{max} > \sigma_{Initial}$ the bank equityholder would tolerate an increase in the level of asset risk up to the point that maximizes her position, σ_{max}. Therefore, the bank would set the upper bound on asset risk at this level. Since the position of the corporation's equityholder increases with asset risk the result would be a shift in the level of risk from its initial level to the level of σ_{max}. □

Risk shifting only occurs on the following two conditions: (1) the value of assets is below some lower threshold, V^*, and (2) given value of assets, the initial level of asset risk is below the level that maximizes the position of the bank's stockholder, σ_{max}. This is in contrast to the basic agent theory, developed by Jensen and Meckling [1976], in which risk shifting may occur in any state.

When the value of the corporation's assets is below the threshold, V^*, the bank's equity value is maximized with a positive level of asset risk. However, risk shifting would occur only if asset value is below a second threshold, V^{**}. As proved in Appendix A.1, when the value of the corporation's assets equals this threshold, the risk that maximizes the position of the bank's equityholder is equal to the initial level of asset risk, $\sigma_{Initial}$. We define V^{**} as:

$$V^{**} \equiv (F^B F^C)^{\frac{1}{2}} e^{-\left(r+\frac{\sigma^2}{2}\right)(T-t)}. \tag{15}$$

Note that V^* is above V^{**} whenever assets are risky. Also, both V^* and V^{**} depend on the geometric mean of the face values of the bank's debt and the corporate's debt. Since the face value of the bank's debt is lower than that of the corporation's debt, both thresholds are lower than the face value of the corporation's debt. Thus, when the value of the corporation's assets crosses these thresholds the corporation is already in financial distress.

Proposition 2. *If the value of the corporation's assets is below the threshold V^{**}, as defined in Equation 15, the asset risk would be shifted from its initial level, $\sigma_{Initial}$, to the level that maximizes the position of the bank's stockholder, σ_{max}, as defined in Equation 14.*

Proof. As shown in Appendix A.1, when the corporation's asset value is below V^{**} we find that $\sigma_{max} > \sigma_{Initial}$ and, therefore, as proved in Proposition 1, the corporation's asset risk would increase to σ_{max} which is higher than the initial risk. □

While we show that a necessary condition for risk shifting is asset value below the lower threshold, V^*, we did not relate this to the effect of leverage. Therefore, as in Merton [1974], we define the quasi leverage ratio as $LR = \frac{F}{V} e^{-r(T-t)}$. We can now express the lower threshold for risk shifting, which appears in Equation 13, in terms of the bank's and corporation's leverage ratios. The value of the corporation's assets is equal to the lower threshold for risk shifting when the geometric mean between the bank and the corporation leverage ratio equals one: $\sqrt{LR^B LR^C} = 1$. Thus, the bank's equityholder might be motivated to increase asset risk only if the corporation is in financial distress.

The expression for the level of asset risk that maximizes the position of the bank's equityholder can be expressed in terms of leverage by rearranging Equation 14:

$$\sigma_{max} = \sqrt{\frac{1}{T-t}\left[ln\left(LR^C\right) + ln\left(LR^B\right) + ln\left(\frac{B^C}{V^C}\right)\right]}. \tag{16}$$

The level of asset risk that maximizes the position of the bank's stockholder increases with its leverage as well as the corporation's leverage. Since increase in asset risk, increases the bank's probability of default and its costs of deposit insurance, our paper supports the advocates for higher capital ratio or lower leverage in banks. The decrease in leverage not only decreases a bank's probability of default due to the higher capital buffer, it also reduces events of risk shifting.

Thus far, we have assumed that by mutual agreement between the creditor and the debtor, asset risk can be shifted to any level. However, risk shifting is often limited by either the available technology or the regulator. Thus, it is interesting to study the equilibrium solution when the level of risk that maximizes the position of the bank's stockholder cannot be reached. In this case, we try to ascertain if a bank's stockholder has incentive to shift risk locally, that is, above its initial level. If at the current level of asset risk the value of a bank's assets is below V^{**}, the bank is motivated to shift asset risk.

Proposition 3. *If the level of the corporation's risk is limited to the domain* $[0, \sigma_{maxposs}]$*, where* $\sigma_{maxposs}$ *is lower than the level that maximizes the position of the bank's stockholder,* σ_{max}*, and if the value of the corporation's assets is lower than* V^{**}*, then risk shifting would occur from its initial level of* $\sigma_{maxposs}$*, to the level of* $\sigma_{Initial}$.

Proof. As proved in Appendix A.1, when $V^C < V^{**}$ the equityholder's position increases with asset risk in the range $\sigma \in [0, \sigma_{max}]$ and decreases in the range $\sigma \in [\sigma_{max}, \infty]$. Since $\sigma_{maxposs}$ is lower than σ_{max} the bank's equityholder would prefer $\sigma_{maxposs}$ over the initial level of asset risk. Since the corporation's equityholder would like to increase asset risk as well, both will agree to increase asset risk to the maximum possible level $\sigma_{maxposs}$. □

4 Bank with Multiple Assets

In this section we extend our model to the case of a bank whose assets consist of two loans to two different corporations. The corporations' asset values follow correlated geometric Brownian motions. We assume the bank can change the terms of the contract of either or both borrowers. Each borrower has no information on the other borrower and her loan contract with the bank.

4.1 The corporations' Liability Structure

The corporation's are funded by equity with a market value of S^i, where $i \in 1, 2$ and a single loan with a face value of F^i and market value of B^i. The loan is a zero-coupon loan maturing at time T and the bank is the sole creditor. The value of each firm's assets, V^i, follows a geometric Brownian motion according to the following equation:

$$dV^i = \mu^i V^i dt + \sigma^i V^i dW^i \tag{17}$$

where μ^i is the instantaneous expected return on the corporation's assets, σ^i is the volatility of the corporation's assets, and dW^i is a standard Wiener process. We denote by ρ the correlation between dW^1 and dW^2.

The event of default occurs at debt maturity, T, if the value of the asset, V_T^i, is lower than the face value of the debt. If default occurs, the creditor takes over the firm without incurring any distress costs and realizes the residual assets of the firm, V_T^i. Otherwise, the debt is fully paid and the creditor, the bank, receives the entire face value of the debt, F^i. The payoff to the corporation's debtholder can be expressed as $B_T^i = \min(V_T^i, F^i)$, or when rearranged:

$$B_T^i = F^i - \max(F^i - V_T^i, 0). \tag{18}$$

As developed by Merton [1974], Equation (18) is equivalent to the payoff of a risk-free debt minus a European put option. Therefore, the present value of the corporate debt is given by:

$$B^i = F^i e^{-rT} - Put(V^i, F^i, \sigma, T) \tag{19}$$

where r is the risk-free rate and $Put(V^i, F^i, \sigma^i, T)$ is the price of a European put option according to the equations developed by Black and Scholes [1973].

As the equity is the residual claim, its payoff at debt maturity is:

$$S_T^i = \max\left(V_T^i - F^i, 0\right). \tag{20}$$

The value of the corporation's stock prior to debt maturity can be replicated by a European call option on the value of the corporation's assets, with a strike price equal to its face value of debt (Galai and Masulis [1976]):

$$S^i = Call(V^i, F^i, \sigma^i, T). \tag{21}$$

where $call(V^i, F^i, \sigma^i, T)$ is the price of a European call option as developed by Black and Scholes [1973].

Since the borrower's equity is a call option on the borrower's asset value, the borrower's equityholders would want to increase risk as much as possible. We assume the borrowers are unaware of each-other and therefore the risk preferences of a borrower's equityholder is unaffected by the asset value, debt value, equity value or risk of other borrowers.

4.2 The Bank's Liability Structure

The bank is funded by equity with a market value of S^B and zero-coupon debt with a face value of F^B and market value of B^B. It is assumed that the debtholders are many small depositors, who are unwilling or unable to monitor the bank manager's actions. Therefore, the bank's stockholder is the only claimholder who controls the level of the debtor's asset risk. As discussed in Marcus and Shaked [1984], due to the periodic frequency of supervisory audits, bank deposits are analogous to a debt claim with an effective maturity equal to the examination interval which we assume to be T.

In the case that the loans portfolio of the bank is composed of N loans to different corporations we can express the value of its assets at debt maturity as the sum of these loans:

$$V_T^B = \sum_{i=1}^{N} F^i - \sum_{i=1}^{N} \max(F^i - V_T^i, 0). \tag{22}$$

The payoff at maturity to the bank's debtholder is the minimum between the value of the bank's assets and the face value of the bank's debt, F^B, and can be expressed as:

$$B_T^B = \min\left(V_T^B, F^B\right) = F^B - max\left[F^B - \sum_{i=1}^{N} F^i + \sum_{i=1}^{N} \max(F^i - V_T^i, 0), 0\right] \tag{23}$$

As the bank's equityholder is the residual claimant, its payoff at maturity is $S_T^B = max[V_T^B - F^B, 0]$. If the bank is solvent at maturity, the equityholder receives the payoff $\sum_{i=1}^{N} F^i - F^B$, which is the maximum payoff that the bank's equityholder can receive. This differs from the basic structural approach in which the equityholders' payoff is unbounded.

The bank equityholder's payoff at maturity can be rearranged and expressed as:

$$S_T^B = S_T^B = max\left[\sum_{i=1}^{N} F^i - F^B - \sum_{i=1}^{N} \max(F^i - V_T^i, 0), 0\right] \tag{24}$$

5 Quantitative Analysis of Bank Risk Shifting

The model developed in the previous sections allows us to examine quantitatively the agency problem between the bank's equityholder and debtholder and its effect on the bank's choice of risk. In this section, we investigate numerically the equilibrium asset risk and its effects on the value of the bank's claims by calibrating our model to typical bank data. Moreover, we consider the effects of risk shifting on a bank's risk-neutral probability of default and its cost of deposit insurance. We first describe the choice of the base case parameters, then focus on the level of asset risk, which is set under different capital structures and asset values.

In our base case, we consider a bank and a corporation with a single debt claim. The debt matures in one year, $T = 1$, following Marcus and Shaked [1984] and Ronn and Verma [1986]. The one-year maturity is justified by the yearly frequency of regulatory audits. If an audit finds the market value of assets to be lower than the value of all liabilities, regulators have the ability to seize the bank. We consider an initial level of asset volatility, $\sigma_{Initial}$, of 15%, similar to the risk of investment grade bonds (Huang and Huang [2012]). The face value of the debtor's debt is $F^C = 80$, and that of the bank's debt $F^B = 73.6$. We assume that all claims are initially fairly priced, which renders the bank's book-value debt-to-asset ratio 92%. Later, we decrease this ratio to 85%, in line with the approach of Admati et al. [2011]. The annual risk-free rate, r, is set at 1%.

Given the recent debate over the necessity and desirability of bank's high leverage relative to that of non-financial corporations (Admati et al. [2011]), we focus especially on the effects of leverage on the level of asset risk, and consequently, on the bank's costs of deposit insurance. We analyze both the effect of a bank's capital structure on risk shifting and the effect of an exogenous change in the value of the corporation's assets on the equilibrium level of asset risk.

Using the base case parameters, we find that, for any asset value below $V^* = 76$, there is a positive level of asset risk, which maximizes the bank stockholder's position. Thus, risk shifting may occur only if the value of the debtor's assets is below this level (Proposition 1). When the debtor's initial level of asset risk is $\sigma_{Initial} = 15\%$, risk shifting occurs at any level of assets below $V^{**} = 75.1$ (Proposition 2).

For example, when asset value decreases to the level of $V^C = 74$, the value of the bank's assets is $V^B = 71.6$ and the value of its equity and debt are $S^B = 2.5$ and $B^B = 69$, respectively. The bank's default probability and cost of deposit insurance are 48.9% and 5.2%, respectively. However, in the case of this asset value, the position of the bank's equityholder is maximized at a risk level of $\sigma_{max} = 22.9\%$.

Consequently, in equilibrium, asset risk would be shifted from its initial level of 15% to this level. When we account for risk shifting, the values of the bank's assets and debt decline to 69.3 and 66.7, respectively. However, in this case the stockholder is better off because the value of the stock increases to 2.6 due to risk shifting.

The bank's probability of default increases to 51.9% and the cost of deposit insurance is 8.4% per unit of the debt's face value. These new levels reflect an increase of 6% in the risk-neutral probability of default and 60% in the value of deposit insurance per dollar of deposits. Since risk shifting occurs when the value of the debtor's assets is already below the face value of the debtor's debt, the bank's probability of default is moderately affected. However, the increase in asset risk affects the depositor's recovery rate even in states of default. Therefore, risk shifting substantially affects the cost of deposit insurance.

5.1 The Effect of a Bank's Capital Structure on Risk Shifting

Following the severe financial crisis of 2007-2009 and the European debt crisis, Admati et al. [2013], Miles et al. [2013] and Turner [2010] claimed that banks' fragility and high probability of default can be remedied by a substantial increase in banks' capital ratio. Thus, we analyze a case in which a bank's leverage is substantially lower than in our base case. The bank's debt-to-asset ratio is equal to the ratio between the face values of the bank's and debtor's debts. Since initially the two are fairly priced, the analysis captures the effect of a bank's capital ratio on risk-taking decisions.

We set the debt-to-asset ratio at 85% by changing the face value of the bank's debt to 68 instead of 73.6. In this case, for any asset value below $V^* = 73$, there is an internal level of asset risk, which maximizes the position of the bank's stockholder. This level is lower than 76, which was found when the debt-to-assets ratio was 92%. Moreover, assuming that the initial level of the debtor's asset risk of is 15%, risk shifting occurs at any level of the debtor's assets below $V^{**} = 72.2$. Consequently, in contrast to the base case, risk shifting would not occur when the asset value decreases to 74. In this case, the value of the bank's assets is 71.6, the value of equity is 5.9 and the value of its debt is 65.5. The bank's probability of default and its cost of deposit insurance are only 28.9% and 2.5%, respectively. The probability of default decreased due in part to the greater equity buffer and in part due to the lower motivation for risk shifting.

The numerical example, which is also proved analytically in Section 3.2, shows that, as the bank's equity layer increases, the stockholder's motivation to shift risk decreases. The result strengthens the argument for a higher capital adequacy in banks, since increasing a bank's capital not only affects its probability of default due to the higher capital buffer, but also reduces the stockholder's motivation to take risk.

5.2 The Effect of Severe Market Conditions on Risk Shifting

Studying the effect of an exogenous decline in the value of a debtor's assets on a bank's resilience is important for the stress test analysis, where the solvency and the ability of a bank to function under severe market conditions are challenged. We focus on a scenario in which the initial value of the debtor's assets is $V^C = 121$. Therefore, its risk-neutral probability of default is relatively low at 0.05% and the cost of deposit insurance is close to zero. In this case, the value of the bank's assets is 79.2 and the value of its equity and debt are 6.3 and 72.9, respectively. The debt is almost riskless since the yield spread over the risk-free rate is close to zero.

We introduce a severe decrease in the value of the debtor's assets by shifting it to 73 (a decrease of 2-3 standard deviations). The result would cause a decrease in the value of the bank's assets to 70.9. The value of the equity and debt are now 2.3 and 68.6, respectively.

This decrease in asset value affects the conditions of the bank - the probability of default increases to 52.5% and the cost of deposit insurance is 5.8% per unit of debt. However, this scenario only partly describes the impact of the decrease in asset value because it ignores risk shifting. When the asset value is 73, the equilibrium level of asset risk is 28.2%. Consequently, the value of the bank's assets decreases to 67.2 and the value of debt equals 64.7. The value of the stock increases to 2.5 due to the positive effect of risk shifting. The higher level of asset risk moderately increases the bank's probability of default to 55.4%, and the cost of deposit insurance almost doubles to 11%.

5.3 The Effect of A diversified loan Portfolio

In most of the literature of the structural approach for pricing corporate liabilities the basic assumption is that bank assets are composed of a single loan. However, in practice bank's assets are composed of a diversified portfolio of individual loan with different asset risk and correlation which is less than one. An exceptional is Flannery [1989] who considers "the properties of individual assets within the portfolio. The analysis highlights the ambiguous impact that increased individual loan risks has on the value of a bank's deposit insurance. Chen et al. [2006] extended and elaborated Flannery [1989] framework by analyzing the cost of deposit insurance for the case of bank's assets which are a portfolio of risky loans.

In this section we numerically study the effect of diversification on risk taking motivation. To keep thew discussion simple and to give the reader the basic intuitions we focus on the case of two assets with a correlation initally of 0.6. To be consistent with the base case parameters, where bank assets are composed from a single loan, we keep the total value of the corporation equals to 76, where the value of each assets is 37 and the face value of each debt instrument is 40. The results are presented at Table (2). When the bank's assets are diversified between the two corporations the risk taking motivation decreases and the value of the bank's stock is maximized at a level of 13.4%, a level which is below the level of 22.9% that maximizes the position of the stockholder in the case of a bank with a single loan. Moreover, under the assumption of initial level of risk of 15%, risk shifting would not occur and consequently the probability of default and costs of deposit insurance are not affected.

The more striking result is presented in Table (3) where the value of one of the borrowers is constant and equal to 35 and the value of the second one receives different levels between 40 and 15. As expected, as the value of the second borrower deteriorate the level of asset risk in equilibrium increases. The level of asset risk of the first borrower is almost constant for different asset value of the second borrower. However, when the value of the second borrower is low enough, the bank is motivated to shift the risk of the first borrower, whose assets are constant. Thus, for a given leverage ratio, the level of risk of a borrower may increase if the quality of the bank's entire portfolio deteriorates.

6 Conclusion

We present a framework that takes into account that bank assets are risky debt claims with limited upside and analyze the risk-taking behavior of banks by considering the strategic relationship between the debtor and creditor. We find that, in equilibrium, the level of asset risk may increase only when the debtor is in financial distress. This result contrasts with the basic agent theory (Jensen and Meckling [1976]), where the equityholder is motivated to increase risk in both solvent and insolvent states of the debtor.

We contribute to the debate over the optimal bank capital ratio by showing that bank stability increases when its capital ratio increases. While recent papers show that increasing bank capital decreases the events of costly default and the costs of deposit insurance (Admati et al. [2013]; Miles et al. [2013]; Turner [2010]), we prove that a higher capital ratio can affect bank stability through a second channel, that is, its negative effect on the equityholders' risk-taking motivation.

By calibrating our model to typical market data, we show that not accounting for the possibility of risk shifting in bad economic states may lead to severe underestimation of the cost of deposit insurance. However, since risk shifting when a bank's assets are risky debt claims is limited to states in which the debtor is already in distress, the effect of risk shifting on a bank's probability of default is relatively minor. In most of the literature of the structural approach for pricing corporate liabilities the basic assumption is that bank assets are composed of a single loan. However, in practice bank's assets are composed of a diversified portfolio of individual loan with different asset risk and correlation which is less than one. When we study the effect of diversification, we find that a diversified loans portfolio decreases risk shifting. Moreover, the optimal level of asset risk decreases as the portfolio is less correlated. Furthermore, we show that for a given leverage ratio, the level of risk of a borrower may increase if the quality of the bank's entire portfolio deteriorates.

Appendix A: Proofs

A.1 Bank Equityholder's Position

The payoff of the bank's equityholder is equivalent to a portfolio of two call options on the value of the corporation's assets (Equation 10). To find the preferred asset risk of the bank's equityholder we calculate the derivative of the value of equity with respect to asset risk:

$$\frac{\partial S^B}{\partial \sigma} = \frac{\sqrt{T-t}}{\sqrt{2\pi}} V^C e^{-\frac{1}{2}\frac{1}{\sigma^2 T}} \left[e^a - e^b\right] \tag{A.25}$$

Where:

$$a = -2\ln V^C \ln F^B + (\ln F^B)^2 - 2\ln\left(F^B\right)\left(r + \frac{\sigma^2}{2}\right)(T-t)$$

$$b = -2\ln V^C \ln F^C + (\ln F^C)^2 - 2\ln\left(F^C\right)\left(r + \frac{\sigma^2}{2}\right)(T-t)$$

There is a constrained maximum for the bank equityholder's position with respect to asset risk in cases where the first derivative equals zero. There are two solutions for the equation when $V^C = 0$ or when $a = b$. By using the terms of the equation $a = b$ we find the corporation's asset value, which maximizes the equityholder's position as a function of asset risk:

$$V^C = (F^B F^C)^{\frac{1}{2}} e^{-\left(r+\frac{\sigma^2}{2}\right)(T-t)} \tag{A.26}$$

Based on Equation A.30, we define V^{**} as the value of assets in which the equity value is maximized for a chosen level of asset risk σ. We note that the derivative changes its sign from positive to negative above the threshold, V^{**}, meaning that the bank's equityholder would like to increase risk below that level and to decrease it above that level of assets.

By using the same equation where $a = b$ we can now fix the level of assets to find the level of asset risk that maximizes the value of the bank's stock:

$$\sigma_{max} = \sqrt{\frac{1}{T-t}\ln\left(\frac{F^B F^C}{(V^C)^2}\right) - 2r} \tag{A.27}$$

However, for both Equations A.26 and A.27 to hold - that is, for an internal solution to exist - the corporation's asset value must be below V^* defined as: $V^* \equiv (F^B F^C)^{\frac{1}{2}} e^{-r(T-t)}$.

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Table 1: Equilibrium outcomes

The base case parameters at the initial state				
F^C	F^B	$\sigma_{Initial}$	r	T
80	73.6	15%	1%	1

Outcome variables										
V^C	V^B	E^B	B^B	V^*	V^{**}	σ_{max}	σ_{UBound}	σ^*	PD^B	$DIPD$
Low asset value with no risk shifting										
74	71.6	2.5	69	76	75.1	22.9%	15%	15%	48.9%	5.2%
Low asset value with risk shifting										
74	69.3	2.6	66.7	76	75.1	22.9%	22.9%	22.9%	51.9%	8.4%

The table presents the set of parameters and payoffs in equilibrium as well as the bank's probability of default and cost of deposit insurance for the case in which risk shifting is impossible and for the case in which risk shifting can take place. The base case parameters F^C, F^B, σ, r and T are the face values of the corporation and bank's debts, the corporation's asset risk, the risk-free rate and the time to maturity of the debt instruments.
The positions in equilibrium V^C, V^B, E^B and B^B are the values of the corporation's asset and of the bank's assets, equity and debt. The thresholds for risk shifting are V^*, the level of assets below which the position of the bank's stockholder is hump-shaped, as defined in Equation 13, and V^{**}, the threshold under which the bank equityholder would like to increase asset risk. The level of asset risk that maximizes the bank equityholder's position, as defined in Equation 14, is σ_{max}, and the upper bound on corporate asset volatility set by the bank's equityholder and the equilibrium asset volatility as defined in Section (3) are σ_{UBound} and σ^*, respectively. The bank's probability of default, as defined in Equation 11, is PD^B and the value of the bank's deposit insurance per dollar of insured deposits, as defined in Equation 12, is $DIPD$.

Table 2: Equilibrium outcomes - Multiple Loans

The base case parameters at the initial state					
F^i	F^B	$\sigma^i_{Initial}$	ρ	r	T
40	73.6	15%	0.6	1%	1

Outcome variables									
V^i	V^B	σ_{V^B}	E^B	B^B	σ^i_{max}	σ^i_{UBound}	σ^{i*}	PD^B	$DIPD$
37	71.55	13.4%	2.11	69.44	13.13%	15%	15%	50.1%	4.7%

The table presents the set of parameters and payoffs in equilibrium as well as the bank's probability of default and cost of deposit insurance for the case of two assets. In this case the borrowing corporations are identical: the face value of debt of both corporations is F^i and initial asset risk of both is $\sigma^i_{Initial}$. The base case parameters F^B, ρ, r and T are the face value of the bank debts, the correlation between the asset values of the corporations, the risk-free rate and the time to maturity of the debt instruments.
The positions in equilibrium V^i, V^B, E^B and B^B are the values of the corporations' assets and of the bank's assets, equity and debt. σ_{V^B} is the risk of the bank's asset portfolio. The level of asset risk that maximizes the bank equityholder's position is found using simulations σ_{max}, and the upper bound on corporate asset volatility set by the bank's equityholder and the equilibrium asset volatility as defined in Section (3) are σ_{UBound} and σ^*, respectively. The bank's probability of default PD^B and the value of the bank's deposit insurance per dollar of insured deposits, $DIPD$ are found using simulations. The number of simulations used is one million.
When using the aggregated method the sum of face values of the banks borrowers is 80 and the sum of the bank's borrowers assets is 74, this is comparable to the case described in Table 1.

Table 3: Equilibrium outcomes - Multiple Loans

The base case parameters at the initial state								
F^1	F^2	F^B	$\sigma^1_{Initial}$	$\sigma^2_{Initial}$	ρ	r	T	V^1
40	40	73.6	15%	15%	0.6	1%	1	35

	V^2					
	40	35	30	25	20	15
σ^1_{max}	27%	31.7%	31.7%	31.7%	31.9%	32.3%
σ^2_{max}	0%	31.7%	63.3%	87.3%	109.8%	133.3%
V^B	72.5	64.5	57.5	52.27	47.56	43.2
E^B	2.48	1.53	1.17	0.9	0.68	0.47
B^B	70	62.96	56.47	51.37	46.89	42.77
PD^B	47.93%	67.76%	76.6%	82.41%	87.08%	91.06%
$DIPD$	3.93%	13.6%	22.51%	29.5%	35.66%	41.31%

The table presents the set of parameters and payoffs in equilibrium as well as the bank's probability of default and cost of deposit insurance for the case of two assets. Throughout the table the value of one borrower's assets remain constant, $V^1 = 35$, while we change the value of the second borrower's asset, V^2. The base case parameters F^1, F^2, F^B, ρ, r and T are the face values of the two borrowers and the bank's debts, the correlation between the asset values of the borrowers, the risk-free rate and the time to maturity of the debt instruments. The positions in equilibrium V^B, E^B and B^B are the values of the bank's assets, equity and debt. The levels of asset risk that maximizes the bank equityholder's position, σ^1_{max} and σ^1_{max}, are found using simulations. The bank's probability of default PD^B and the value of the bank's deposit insurance per dollar of insured deposits, $DIPD$ are found using simulations. The number of simulations used is one million.

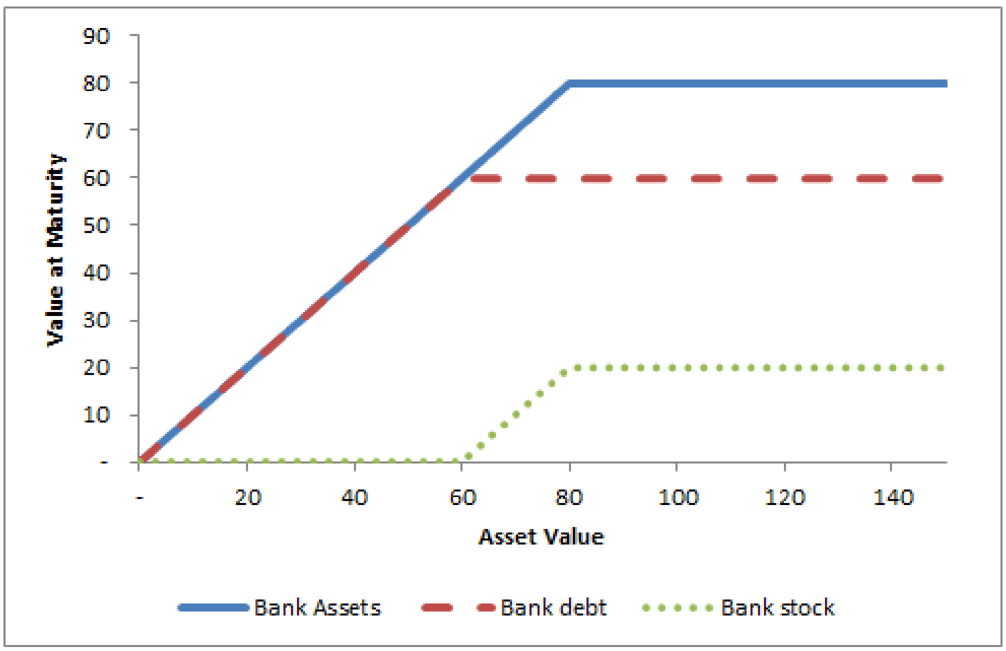

Figure 1: The value of the bank's asset, debt and equity at debt maturity: The values in the figure refer to a bank with a single asset - a corporate loan with a face value of 80 and time to maturity of one year. The bank is financed with equity and a single bond with a face value of 60 that matures in one year.

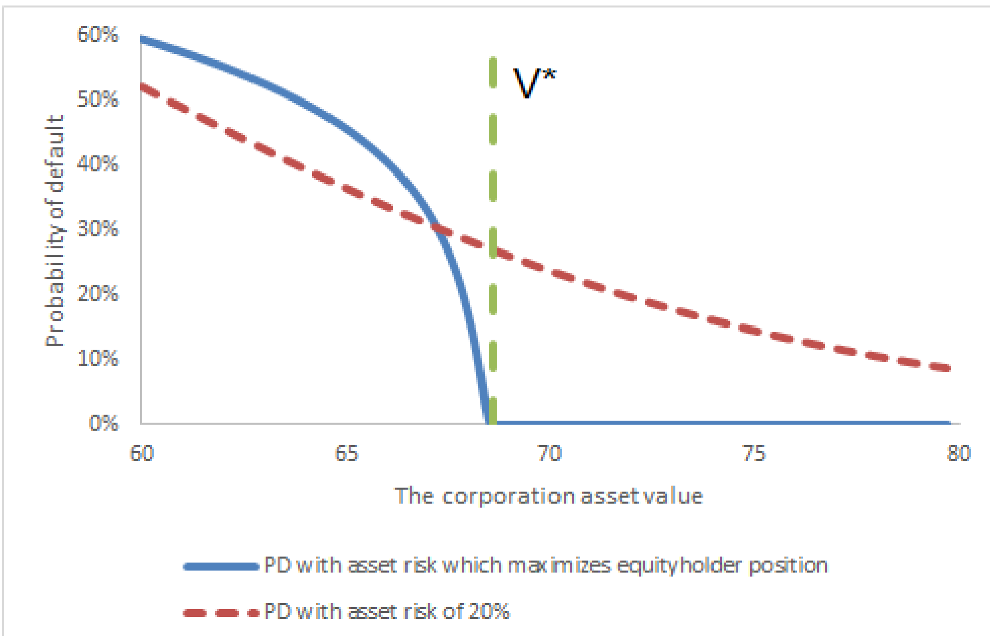

Figure 2: The bank default probability with asset risk that maximizes the position of the bank's equityholder: The values in the figure refer to a bank with a single loan with a face value of 80 and time to maturity of one year. The bank is financed with equity and deposits with a face value of 60 and time to maturity of one year. The risk-free rate is 1%. The dotted line depicts the bank's probability of default when asset risk is constant and equal to the initial volatility of the corporate assets, 20%. The blue line is the bank's probability of default when the value of asset risk is the value that maximizes the bank equityholder's position. The vertical dashed line indicates the threshold for risk shifting, V^*.

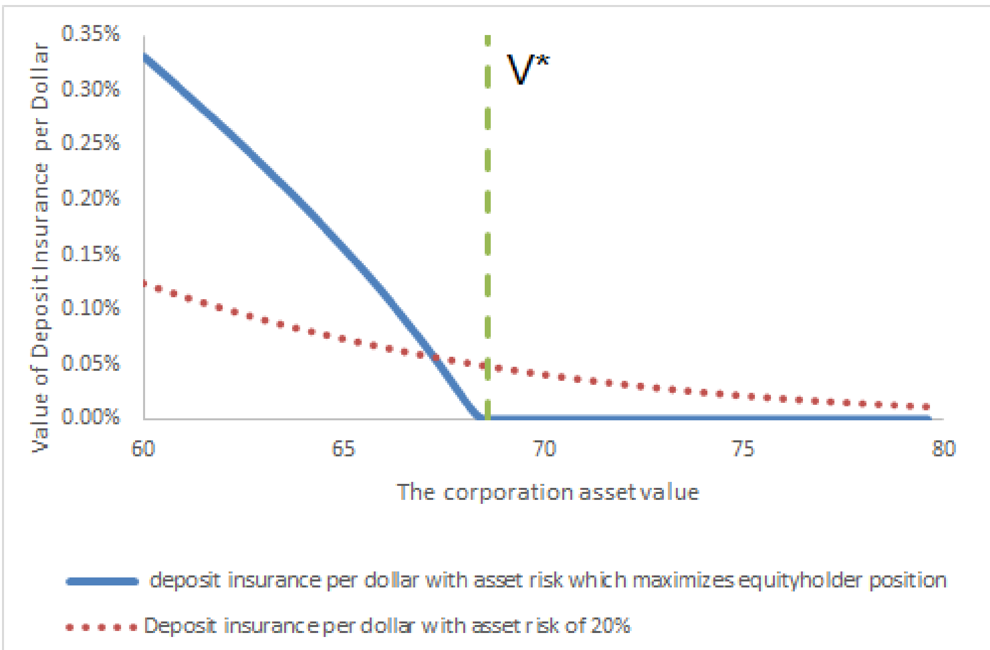

Figure 3: The value of deposit insurance with asset risk that maximizes the bank equityholder's position: The values in the figure refer to a bank with a single loan with a face value of 80 and time to maturity of one year. The bank is financed with equity and deposits with a face value of 60 and time to maturity of one year. The risk-free rate is 1%. The dotted line depicts the value of deposit insurance per dollar of deposits when the corporate asset risk is constant and equal to the initial asset risk of the debtor's assets, 20%. The blue line is the value of the deposit insurance per dollar of deposits when the value of asset risk is the value that maximizes the bank equityholder's position. The vertical dashed line indicates the threshold for risk shifting, V^*.

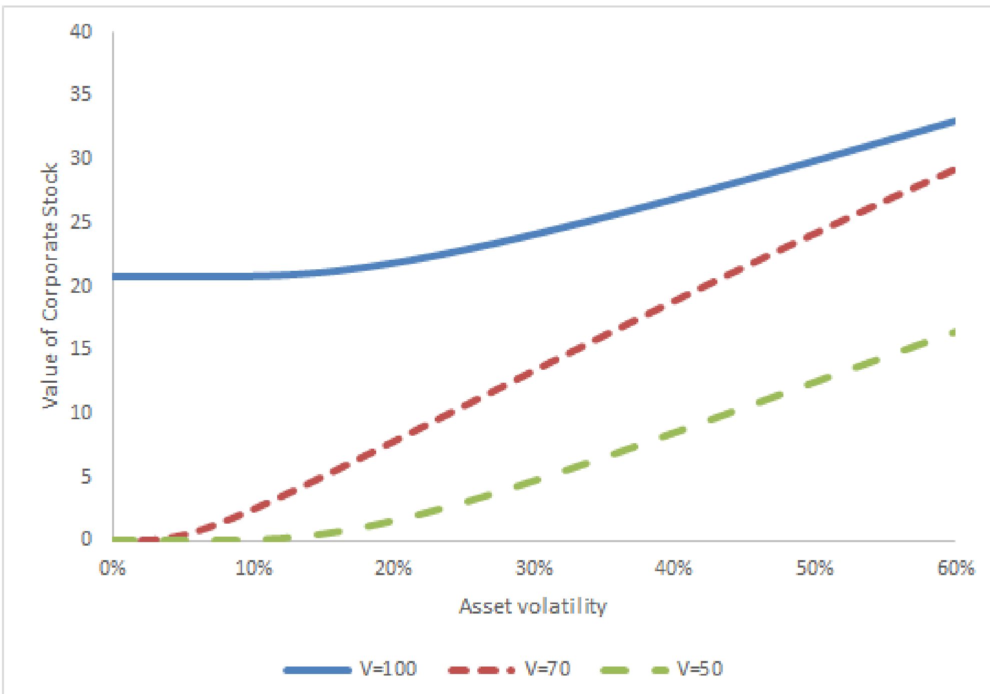

Figure 4: The value of the debtor's stock as a function of its asset risk: The values in the figure refer to a corporation with a single debt instrument with a face value, F^C, of 80 that matures in one year and where the risk-free rate is 1%. We analyze three scenarios where the corporation's asset value is 50, 70 and 100.

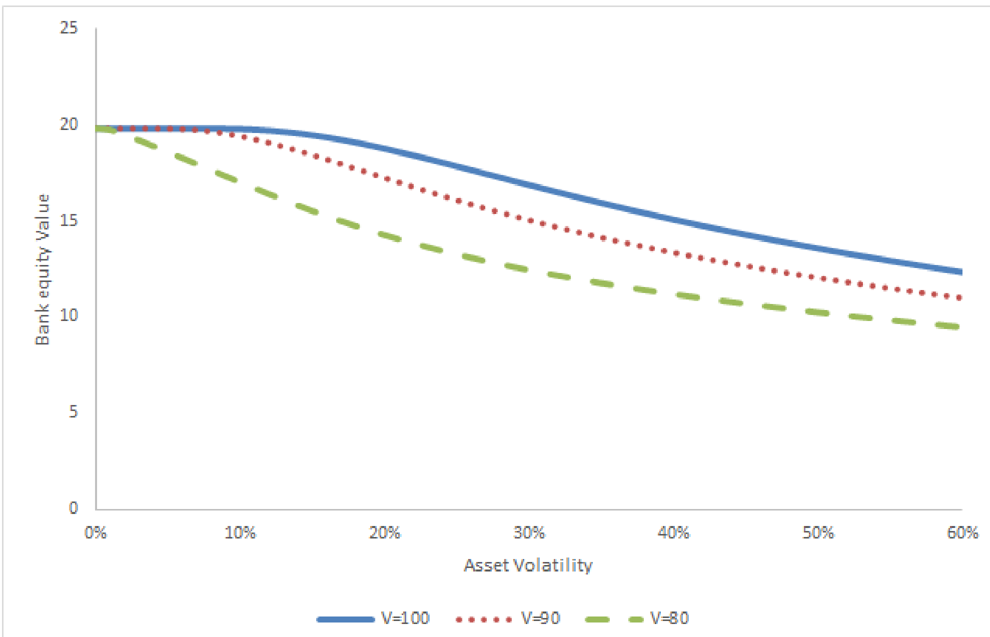

Figure 5: The value of bank equity as a function of the debtor's asset risk The values in the figure refer to a bank with a single loan with a face value of 80 and time to maturity of one year. The bank is financed with equity and a single bond with a face value of 60 and time to maturity of one year. The risk-free rate is 1%. We graph the value of bank equity for three different corporate asset values: 100, 90 and 80. All of these values are above the threshold for risk shifting, V^* which equals 67.2.

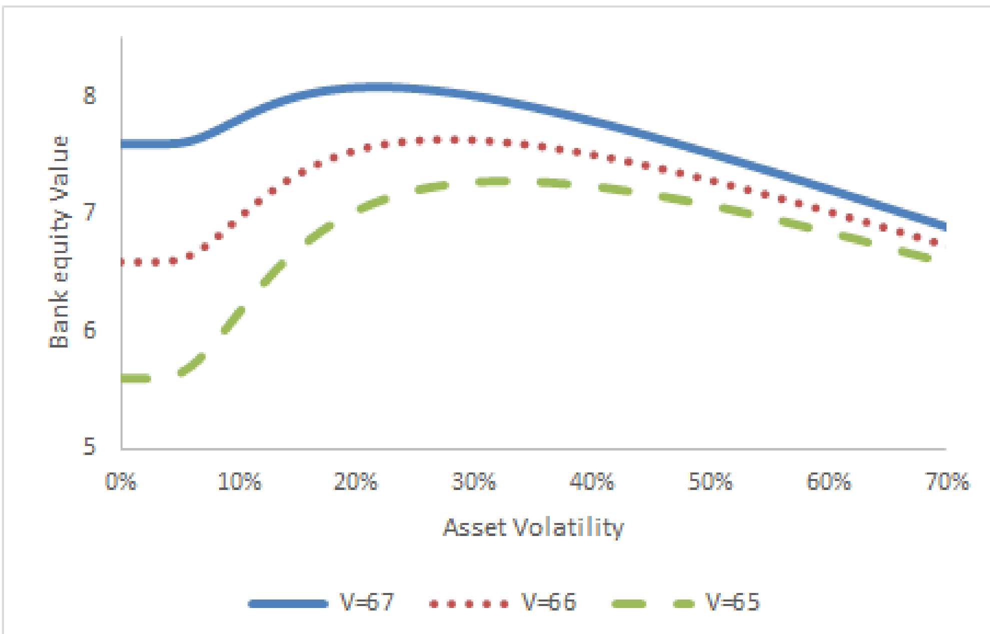

Figure 6: The value of bank equity as a function of the debtor's asset risk: The values in the figure refer to a bank with a single loan with a face value of 80 and time to maturity of one year. The bank is financed with equity and a single bond with a face value of 60 and time to maturity of one year. The risk-free rate is 1%. We graph the value of bank equity for three different asset values - 67, 66, and 65 - all of which are below the threshold of risk shifting, V^*, which is 67.2.

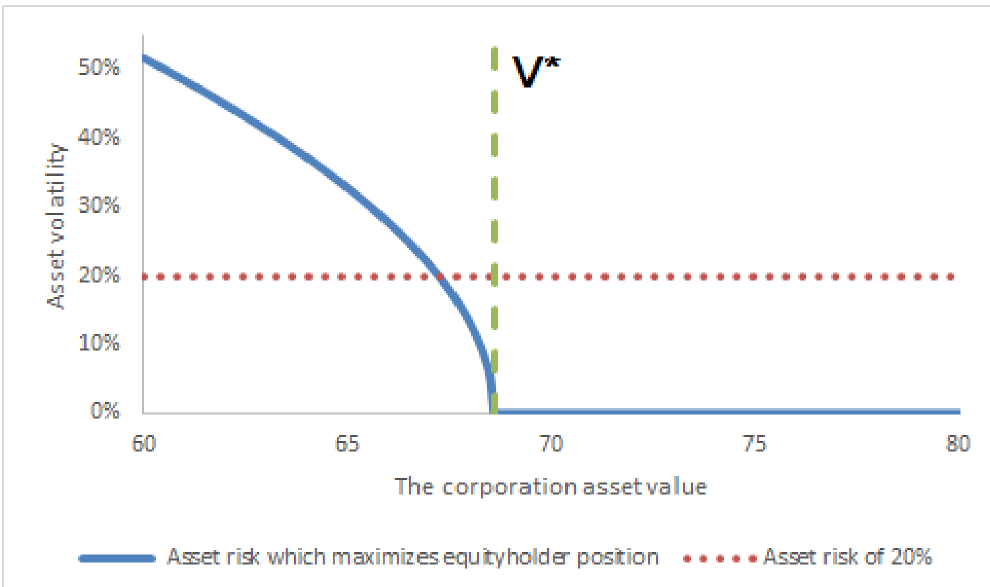

Figure 7: The debtor's asset volatility that maximizes the position of the bank's equityholder: The values in the figure refer to a bank whose assets consist of a single loan with a face value of 80 and time to maturity of one year. The bank is financed with equity and deposits with a face value of 60 and time to maturity of one year. The risk-free rate is 1%. The blue line is the asset risk of the debtor which maximizes the position of the bank's equityholder, and the dotted line is the initial level of asset risk, which equals 20%. The vertical dashed line indicates the threshold for risk shifting, V^*.

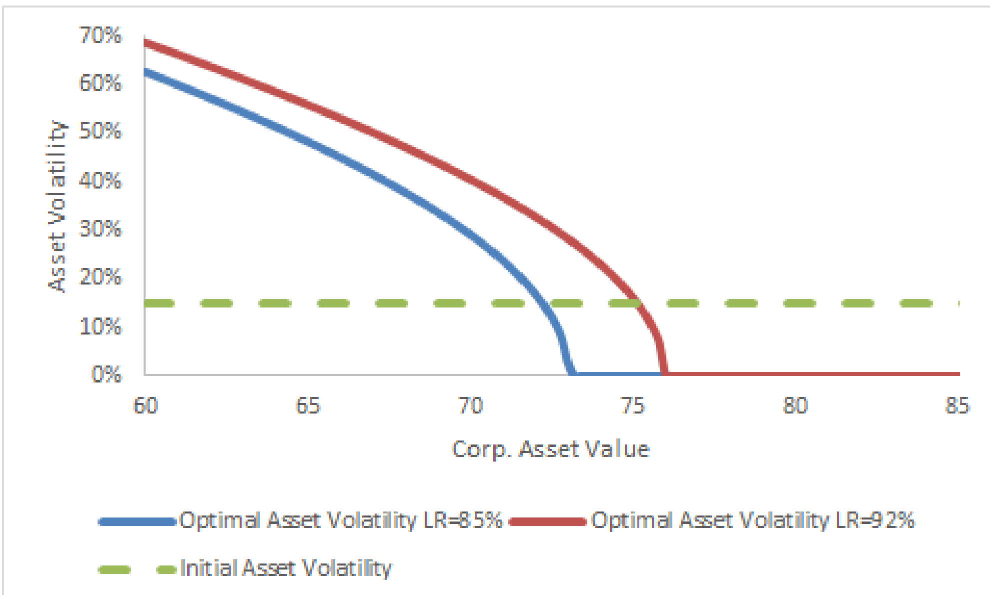

Figure 8: Optimal asset risk for different bank leverage ratios: The values in the figure refer to a bank with a single loan with a face value of 80 and time to maturity of one year. The bank is financed with equity and a single bond with time to maturity of one year. In the first case, the face value of the bond is 73.6, representing a leverage ratio of 92%. In the second case, the face value of the debt is 68, representing a leverage ratio of 85%. The risk-free rate is 1% and the debtor's asset risk, σ, is 15%.