PRELIMINARY VERSION

Funding Risk and Collateralized Lending

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Abstract

This paper uses a stylized 3-action global game banking model to compare the effects of secured and unsecured lending. We establish the following key results. First, the availability of secured wholesale debt exerts a catalytic effect on lending in equilibrium and lowers the bank's funding risk. This leads to an increase in welfare. Second, however, secured financiers may finance inefficient banks. This may lead to a reduction in welfare. Third, we show under which conditions the overall effect from introducing secured debt on expected welfare is positive. Fourth, our model predicts tiering: Sound banks with good fundamentals are more likely to refinancer using unsecured debt, while unsound banks are more likely to issue secured debt.

Keywords: liquidity risk, coordination failure, global games, secured funding

JEL classification codes: G 21, G 23, G 32, C 72, D 82

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1. Introduction

The funding structures of banks matter a great deal for financial stability. In the run-up to the 2008/2009 financial crisis banks had increasingly made use of unsecured wholesale funding, often with rather short maturity, to finance the expansions of their balance sheets. This strong reliance on wholesale debt proved detrimental to financial stability during the crisis when funding markets froze and banks became subject to wholesale debt runs. Since the crisis, however, banks rely to a greater extent on secured debt. This implies that the fraction of assets encumbered for collateralization purposes has considerably risen, while the issuance of unsecured bank debt has declined strongly.[1] By now, the recent trend towards secured debt and higher asset encumbrance is also regarded with suspicion by policy makers and market participants. In particular, it is feared that secured funding may hamper financial stability since a higher level of asset encumbrance may lower financiers' incentives to ensure an efficient liquidation of their encumbered assets in case of default. This may induce a dilution of unsecured financiers' claims. Furthermore, since the claims of secured financiers are seldom touched during a bail–in, a higher level of asset encumbrance reduces the scope for a bail–in in case of an outright default or a government intervention. Besides these detrimental implications for financial stability, however, an offsetting stabilizing effect from the use of secured funding may exist since collateralized debt may be less 'flighty' and more persistent. This may lower banks' exposure to funding risk and, in turn, may also prove beneficial to unsecured wholesale financiers and retail depositors.

In this paper, we present a stylized bank funding model that captures these different aspects. Funding risk is modeled as the risk of a miscoordination among wholesale financiers in the spirit of the global game models by Rochet and Vives (2004) or Goldstein and Pauzner (2005). The novelty of our approach is that we use a three-action global game where financiers face a choice between refusing to provide funds, providing secured funding or providing unsecured funding. By comparing the equilibrium outcome of this model to a benchmark case where only unsecured debt is available, we establish the following results. First, the availability of secured funding reduces coordination and funding risk and therefore increases banks' ability to attract funding. The possibility of attracting secured funding may allow banks to continue financing investments with positive net present value,

[1] See e.g. Houben and Slingenberg (2013) or IMF (2013).

which would otherwise default when unsecured financiers refuse to bear the associated coordination risk. We establish a very simple condition for this *catalytic effect of secured funding* to be present. However, this stablizing effect comes at the expense of retail depositors and unsecured wholesale financiers whose claims (contingent on default) are diluted. As a consequence, whether the availability of secured debt benefits retail depositors depends crucially on the strength of the catalytic effect. While the catalytic effect refers to the general reduction in funding risk following the availability of secured debt, our second result concerns the composition of the debt structure. We provide conditions under which the availability of secured funding can either crowd–in or crowd–out unsecured lending. In particular, when the interest rate spread between unsecured and secured debt is sufficiently large, unsecured debt is relatively more attractive (conditional on the bank being able to refinance itself) and crowding–in occurs. That is, the additional funding triggered by the mere availability of secured debt will be a mix of secured and unsecured debt. Conversely, when the spread is small, unsecured debt is crowded–out, i.e. some previously unsecured funding will be substituted by secured funding. Third, we show that the safety associated with the recourse to the bank's collateral in case of default can induce secured financiers to finance inefficient investments. This result clearly shows that secured funding may be a double–edged sword. Although it reduces funding risk and thereby helps banks to continue investments with positive net present value, it also allows weaker banks to fund assets with negative NPV that should actually be liquidated. This happens because secured financiers obtain safety at the expense of unsecured financiers and depositors, and therefore they may not care about the efficiency of their investments. In sum, the rise in secured funding will improve economic efficiency for banks with more healthy balance sheets, but reduces economic efficiency for unsound banks. Fourth, when banks diversify their funding structure by offering secured and unsecured funding, tiering may be observed: Stronger banks may fund themselves completely through the unsecured market, whereas weaker banks rely on secured funding.

2. Relation to the Literature

Our model belongs to a strand of the literature that views illiquidity as the outcome of a coordination failure on the side of the creditors. This perspective dates back to Diamond and Dybvig (1983). They show that a bank may be in one of two pure strategy Nash equilibria that are sustained by self-fulfilling beliefs. The bank either successfully completes its long-term investments or it fails early due to a depositor run. This equilibrium multiplicity results from the indeterminacy of agents' beliefs (Morris and Shin, 2004). To derive a unique equilibrium in the Diamond-Dybvig model, Goldstein and Pauzner (2005) employ the global game methodology of Carlsson and Van Damme (1993). Compared to the original model, the optimal deposit contract is altered due to the prevalence of strategic uncertainty among creditors that arises from the possibility of a coordination failure. Rochet and Vives (2004) devise a technically more tractable global game where the deposit contract is exogenous but that allows to study the influence of changes in the composition of bank balance sheets on the failure probability and to decompose default risk into liquidity and solvency risk. Vives (2014) and König (2015) further use this framework to study the impact of regulatory capital and liquidity ratios on banks' riskiness. One shortcoming of these models is their focus on binary action banking models where creditors could decide between rolling over into some form of short-term debt or to withdraw funding. In practice, however, banks seldom use a single form of short-term debt but rather different types of debt that can be differentiated along different dimensions such as seniority or degree of collateralization. The novelty of our approach is that we consider a three-action global game that allows us to study the effects of a more differentiated debt structure where creditors can invest in either unsecured or secured debt. To this end we use techniques and heuristics developed in Basteck and Daniëls (2011) and Basteck et al. (2013) for global games with three or more actions. They already show that the introduction of a third action in a standard binary action game with two equilibria may change the selection between those equilibria. Moreover, they also provide an example that suggested that this insight may apply to collateralized lending. Yet, in contrast to the present model, their example was not rich enough to discuss the efficiency and stability of different funding arrangements and to derive policy conclusions. Ahnert et al. (2016) also study the effects of a more differentiated bank liability structure by introducing covered bank bonds into the Rochet-Vives set-up with binary actions. They find that, on the one hand, as covered bonds are a stable funding instrument, re-financing through covered bonds reduces bank default risk. On the other hand, the asset encumbrance associated with the use of covered bonds dilutes the claims of unsecured creditors, thereby raises their incentives to withdraw early and increases default risk.[2] Similarly, the use of secured funding in our model is also

[2]The importance of studying the properties of different funding structures from a financial stability perspective is further highlighted in more policy-oriented work by Gai et al. (2013) and Eisenbach et al. (2014).

only possible because the claims of retail depositors in case of bank default are diluted when assets are pledged as collateral. However, we assume that retail creditors are passive and do not react to the increase in secured funding by withdrawing their deposits. Moreover, in contrast to Ahnert et al. (2016) who focus on the roll-over problem of unsecured creditors, we allow wholesale creditors to invest into unsecured debt or secured debt. This allows us to study the the differential stability and efficiency properties of secured versus unsecured funding in greater detail.

3. Model

We study the funding problem of a bank that operates for two periods $t = 1, 2$. The bank's balance sheet is normalized to unity. The bank can borrow on a financial market from a continuum of risk-neutral wholesale investors of mass m each holding a unit endowment.

3.1. Bank Assets

The bank holds one unit of a risky asset. The asset is illiquid and can be liquidated in $t = 1$ only for a value $\ell < 1$. When held until $t = 2$, it yields a stochastic return

$$\tilde{R} = \begin{cases} R_g & \text{with probability } p(\theta) \\ R_b & \text{with probability } 1 - p(\theta) \end{cases}$$

where $R_g > R_b$. The success probability is described by a continuous and strictly increasing mapping $p : \mathbb{R} \to [0, 1]$. θ parameterizes the state of the economy that is realized at the beginning of $t = 1$. For simplicity, we assume that it is drawn from a uniform prior over a sufficiently large compact support. To make the bank's refinancing problem interesting, we maintain

Assumption 1. $R_g > 1 + r > \ell > R_b$

where r denotes the risk-free interest rate. The first inequality states that, conditional on the state being good, it is efficient to fund the asset. The second inequality states that liquidation is inefficient, whereas the third one states that, conditional on the bad state, it is more efficient to liquidate the asset in $t = 1$ rather than keeping the asset on the balance sheet. As a consequence of Assumption 1, it is efficient to hold the asset until $t = 2$ if and only if

$$p(\theta) \geq p(\theta_{\text{eff}}) \equiv \frac{\ell - R_b}{R_g - R_b} \tag{1}$$

3.2. Bank Liabilities

We abstract from equity and long-term wholesale debt. Accordingly, the bank is financed by short-term wholesale debt $\gamma \in [0,1]$ and retail deposits $1-\gamma$. Retail deposits are fully insured at a flat rate normalized to zero. Thus, the interest rate on retail deposits equals the risk-free rate r. Retail depositors' claims (or the claims of the deposit insurance fund) are senior to unsecured wholesale claims.

Short-term wholesale debt has to be refinanced in $t=1$ at the prevailing interbank rates $r_u < R_g - 1$ (when funding is unsecured), or $r_s \in [r, r_u]$ (when funding is secured with collateral). Limited liability implies that the bank will want to hold on to the asset until $t=2$, independent of the realized $p(\theta)$. We assume that the market is sufficiently liquid such that the bank is potentially able to refinance fully, $m \geq \gamma$ and we further impose

Assumption 2. $\ell < (1-\gamma)$

The assumption implies that unsecured wholesale financiers go away empty-handed in case the project defaults either in periods $t=1$ or $t=2$. The introduction of secured wholesale debt below will de facto alter this seniority structure.

4. Benchmark case: Refinancing with unsecured debt

Assuming that retail investors are passive and do not face any liquidity needs in $t=1$, the bank has to refinance wholesale debt of amount γ. In this section we discuss the case where the bank can refinance by issuing only unsecured one-period wholesale debt in $t=1$ at the interest rate r_u.

4.1. Equilibrium Multiplicity

Whenever $p(\theta)$ is common knowledge among wholesale financiers, the model exhibits multiple self-fulfilling equilibria for $p(\theta) > p(\underline{\theta}) \equiv \frac{1+r}{1+r_u}$, similar to Diamond and Dybvig (1983): In one equilibrium financiers believe that nobody is willing to provide funding, implying that they are better off by investing their unit endowment risk-free at interest rate r. This implies that the bank cannot get funded and defaults in $t=1$ thereby vindicating creditors' initial beliefs. Conversely, if financiers believe that enough other financiers are willing to provide funding, their expected payoff from investing with the bank exceeds the payoff from the risk-free asset since $p(\theta) > p(\underline{\theta})$. As a consequence, all financiers are willing to fund the bank, implying that the initial belief is again vindicated.

4.2. Global Game Solution

To solve for a unique equilibrium, we use standard global game techniques (Carlsson and Van Damme, 1993; Morris and Shin, 2003; Frankel et al., 2003). To this end, we augment the model as follows. First, the true success probability $p(\theta)$ is no longer common knowledge among financiers. While wholesale financiers know the functional form of $p(\cdot)$, they are unsure about its true realization. Thus, each wholesale financier i forms a private belief about about the success probability, based on the observation of a private signal

$$p(x_i) = p(\theta + \sigma \varepsilon_i)$$

where ε_i is i.i.d., independent of θ, and drawn according to a continuous, symmetric density f with unit variance over a compact support. $\sigma \geq 0$ is a scale parameter that will be interpreted below as a measure of bank balance sheet transparency. Financier i then attaches probability $p(x_i)$ to the bank's terminal success.[3] Second, we assume that there exist regions of very high and very low success probabilities, so-called *upper* and *lower dominance regions*, where financiers strictly prefer to fund the bank or refuse to provide funding respectively: For any $p(x_i) < p(\underline{x}) \equiv \frac{1+r}{1+r_u}$ the expected payoff from investing with the bank is less than the risk-free payoff. While this lower bound emerges naturally from the payoff structure, the model must be amended slightly to obtain an upper dominance region. We assume that there exists $\overline{x}$, such that for any $x_i > \overline{x}$ a financier attaches probability $p(x_i) = 1$ to the terminal success of the bank. In addition, we assume that in this case the liquidation value ℓ improves to a value $\overline{\ell} \in (1, R_g)$.[4] Taken together, this implies that financiers with belief $p(x_i) > p(\overline{x})$ will always prefer to invest with the bank.

In this augmented version of the model, a *strategy* for financier i is a mapping that associates a signal realization x_i with an action $a_i \in \{\text{fund, refuse to fund}\}$. A *threshold strategy* is described by a value x_i^* such that financier i refuses to fund if $x_i < x_i^*$ and otherwise provides credit to the bank.

[3]We use this private value formulation of the game, rather than a common value formulation, for analytical simplicity. See also the discussion of assumptions below.

[4]See Goldstein and Pauzner (2005) for a similar modification in the Diamond-Dybvig bank-run model and an exhaustive discussion of the different interpretations.

A *symmetric threshold strategy* is one where $x_i^* = x^*$ for all i. According to standard results in the literature, this modified refinancing model exhibits a unique symmetric threshold equilibrium.

Proposition 1. *(Unique Refinancing Equilibrium with Unsecured Debt.)*

1. *There exists a unique symmetric threshold equilibrium summarized by* $\{x_u^*, \theta_u^*\}$. *Financiers provide unsecured funding if and only if* $p(x_i) \geq p(x_u^*)$, *while the bank is able to refinance with unsecured debt if and only if* $p(\theta) \geq p(\theta_u^*)$.
2. *There are no other equilibria in non–threshold strategies.*
3. *In the limit as* $\sigma \to 0$, $p(\theta_u^*) \to p(x_u^*)$.

Proof. See Morris and Shin (2003). □

It is straightforward to calculate the critical probabilities $p(x_u^*)$ and $p(\theta_u^*)$ explicitly. As financiers use threshold strategies around x_u^*, by the law of large numbers, the fraction of financiers who provide funding for a given θ is given by $\mathbf{Pr}(x_i > x_u^* \,|\, \theta)$. The bank's failure point θ_u^* then solves the following condition

$$\begin{aligned} & \mathbf{Pr}(x_i < x_u^* \mid \theta_u^*) = \frac{m-\gamma}{m} \\ \Leftrightarrow \quad & F\left(\frac{x_u^* - \theta_u^*}{\sigma}\right) = \frac{m-\gamma}{m} \\ \Rightarrow \quad & p(\theta_u^*) = p\left(x_u^* + \sigma F^{-1}\left(\frac{\gamma}{m}\right)\right) \end{aligned} \tag{2}$$

where $F(\cdot)$ denotes the cumulative distribution function of ε_i.[5]

To solve for $p(x_u^*)$, let $q^*(x_i) \equiv \mathbf{Pr}(\theta > \theta_u^* | x_i)$ denote the probability that a financier with signal x_i assigns to the bank being able to obtain enough funds at $t = 1$. Accordingly, the converse probability $1 - q^*(x_i)$ reflects financier i's assessment of the bank's funding or coordination risk.

In equilibrium, financiers are willing to provide unsecured funding if and only if

$$q^*(x_i)p(x_i)(1 + r_u) \geq 1 + r \tag{3}$$

[5]The last line follows by the symmetry of the distribution, $F^{-1}(x) = -F^{-1}(1-x)$.

A financier who observes the threshold signal x_u^* must be exactly indifferent between providing funding to the bank and investing risk-free, implying that Eq. (3) holds with equality at $x_i = x_u^*$. Under the distributional assumptions, the equilibrium funding risk of the bank is given by

$$1 - q(x_u^*) = \mathbf{Pr}(\theta < \theta_u^* \mid x_u^*) = \mathbf{P}(x_i > x_u^* \mid \theta_u^*) = \frac{\gamma}{m} \tag{4}$$

where the first equality holds by definition, the second equality follows from the flat prior assumption and is standard in global games (Morris and Shin, 2003), and the third equality follows from Eq. (**??**) and the definition of θ_u^*. Thus, equilibrium funding risk is entirely governed by the ratio of required wholesale funding of the bank to market thickness m. An increase in γ (decrease in m) increases funding risk as it raises the bank's exposure to a miscoordination of financiers in the market for wholesale debt.

Combining Eqs. (3) and (4) yields

$$p(x_u^*) = \frac{1+r}{1+r_u} \times \frac{m}{m-\gamma} \tag{5}$$

Using Eqs. (2) and (4), it is straightforward to show that for $\sigma \to 0$, the so-called global game solution obtains with $p(\theta_u^*) \approx p(x_u^*)$. In this case, the equilibrium is described by a step function. For $\theta < \theta_u^*$, all creditors refuse to provide funding, whereas for $\theta > \theta_u^*$ all creditors provide unsecured funding.

[Figure 1 here]

4.3. Efficiency Properties of Equilibrium Solution

The funding decisions of financiers are generically inefficient. Let x_{eff} denote the signal such that financiers with signal $x_i \geq x_{\text{eff}}$ believe that the bank holds positive NPV assets and note that $x_{\text{eff}} = \theta_{\text{eff}}$. Financiers provide funding only to tbanks that they believe hold assets with a sufficiently large positive NPV, implying that they never provide funding to banks that they deem to have negative NPV assets. However, there are some banks holding positive NPV assets that are not funded, even though financiers believe these banks to be efficient.

Corollary 1. *(Inefficient Bank Funding)*

1. *Wholesale financiers never fund inefficient banks:* $p(x_u^*) \nless p(x_{\mathit{eff}})$.

2. *Wholesale financiers refuse to refinance some banks even if they consider them to be efficient:* $p(x_u^*) > p(x_{eff})$.

Proof. See Appendix. □

The reason why financiers refuse to fund some banks even though they consider them to be efficient stems from the joint occurrence of two effects. First, *strategic uncertainty* among wholesale financiers implies that $p(x_u^*) > p(\underline{x})$. Second, $p(\underline{x}) > p(x_{\text{eff}})$ for any interest rate $r_u \in [r, R_g - 1]$.[6] That is, there exists no feasible interest rate that can compensate junior wholesale creditors sufficiently enough for the loss of their initial investment in the bad state and could induce them to fund *all* banks with positive NPV assets. That wholesale creditors go away empty-handed in the bad state is, in turn, a consequence of the seniority of retail depositors' claims and Assumption (2).

Even though financiers never provide funding to banks that are deemed to have negative NPV assets, it may still happen that inefficient banks can obtain enough funding. To see this, note first that changes in the degree of balance sheet transparency σ can exert an ambiguous effect on the bank's failure point.

Corollary 2. *(Balance Sheet Transparency)*
If the bank's balance sheet becomes less transparent (σ increases), the banks' failure point increases (decreases) if and only if the coordination risk γ/m is larger (smaller) than $1/2$.

Proof. See Appendix. □

The intuition behind Corollary 2 is straightforward. Given the symmetry of the noise distribution $F(\cdot)$, the bank's failure point and the financiers' threshold must be equal whenever financiers believe that both events, the bank obtaining funding and the bank becoming illiquid, occur with equal probability, i.e. if $\gamma/m = 1/2$. If, however, $\gamma/m < 1/2$, the failure point θ_u^* must lie to the left of x_u^*. Moreover, by the flat prior assumption, a decline in balance sheet transparency (increase in σ) leaves the midpoint of the distribution unchanged but shifts probability mass to its tails. Hence, the bank can survive for a larger range of states θ and the failure point θ_u^* declines. This is illustrated in Fig. xx.

As a consequence of Corollary 2, if financiers' information is rather imprecise and funding risk is sufficiently small, the bank's failure point may even come to lie below the efficiency point θ_{eff}.

[6] The unsecured interest rate that equates $\underline{\theta}$ and x_{eff} is given by $1 + \frac{(R_g - \ell)(1+r) - rR_b}{(1+r)\ell - R_b} > R_g$.

Corollary 3. *(Inefficient Continuation)*

Even banks with negative NPV assets can obtain funding, i.e. $\theta_u^* < \theta_{eff}$*, if and only if the degree of transparency is sufficiently small (i.e.* σ *sufficiently large) and the coordination risk is sufficiently small (i.e.* $\frac{\gamma}{m} < \frac{1}{2}$*).*

Proof. See Appendix. □

Ceteris paribus, financiers are more willing to provide funding when the coordination risk γ/m is small. But then, they may accidentally provide funding to inefficient banks and allow them to continue whenever their information is sufficiently imprecise and $\theta_u^* < \theta_{\text{eff}}$. In the limit however, when balance sheet transparency is high and financiers' information becomes very precise, $\sigma \to 0$, Corollary 3 always fails to hold. In this case, inefficient banks never receive funding and also some efficient banks fail to survive since $\theta_u^* \approx x_u^* > \theta_{\text{eff}}$.

5. Equilibrium with Secured and Unsecured Debt

We next introduce the possibility that wholesale creditors can invest into two different types of wholesale debt, unsecured debt (as above) and collateralized debt which is secured by pledging the bank's asset as collateral. Although retail depositors – or for that matter, the deposit insurance fund – are assumed to have seniority over unsecured wholesale creditors in case of default, this seniority structure is implicitly altered once the bank puts up its asset as collateral to secure wholesale debt. Given that the deposit insurance premium does not reflect the particular type of wholesale debt that the bank offers, the issuance of secured wholesale debt exerts a negative externality on retail depositors. In fact, it is because of the possibility of diluting retail creditors' claims that the bank can offer fully secured wholesale claims; the larger the fraction of retail deposits, the easier it becomes to guarantee a full repayment of secured wholesale creditors' investments in the bad state. To highlight this, we maintain the following

Assumption 3. $\gamma \leq R_b$

Secured wholesale debt then pays out $1 + r_s$ in $t = 2$ when the good state occurs. In case of default at either $t = 1$ (due to a coordination failure) or $t = 2$ (when the bad state occurs), wholesale creditors receive the respective liquidation value of the asset up to their initial unit investment. Given Assumption 3, this implies a safe unit payoff in case of default.[7]

5.1. Characterization of Equilibrium

A strategy in this extended model is given by a mapping from financier's signal x_i into actions $a_i \in$ {fund unsecured, fund secured, refuse to fund}. Without loss of generality, we assume that the actions are ordered as follows

$$\text{refuse to fund} \prec \text{fund secured} \prec \text{fund unsecured}$$

Given this ordering, it can be easily checked that for each θ creditor i's payoff difference between any two actions (weakly) increases in the action chosen by others, implying that the extended game with three actions is a game of strategic complementarities (or a supermodular game). Moreover, the game exhibits state monotonicity, i.e. the payoff difference between any two actions increases in θ, implying that higher states make higher actions more appealing.

As in the benchmark model with unsecured funding, the equilibrium in this extended model will be monotone, i.e. the mapping from states θ to actions a_i can be characterized by an increasing step function. To derive the equilibrium, we begin by applying the reasoning from the benchmark model to the choice between refusing to fund and secured funding. Suppose there exists θ_s^* such that the bank fails if and only if $\theta < \theta_s^*$. Similar to Eq. (3), a financier prefers secured wholesale debt compared to the risk-free asst if and only if

$$q^*(x_i)p(x_i)(1+r_s)+(1-q^*(x_i)p(x_i)) \geq 1+r \quad \Leftrightarrow \quad r_s q^*(x_i)p(x_i) \geq r \tag{6}$$

which holds with equality at the threshold x_s^*. Since the failure condition of the bank remains unchanged, $q^*(x_s^*) = 1 - \frac{\gamma}{m}$. The critical probability that a financier needs to assign to become willing to switch from no lending to secured lending is given by

$$p(x_s^*) = \frac{r}{r_s} \times \frac{m}{m-\gamma} \tag{7}$$

Using Eq. (2), the bank's new failure point θ_s^* becomes

$$\theta_s^* = x_s^* + \sigma F^{-1}\left(\frac{\gamma}{m}\right) \tag{8}$$

[7] The payoffs for this extended game are shown in Table **??** in the Appendix.

Next, consider the choice between secured and unsecured funding. Financiers will prefer to invest in unsecured debt if and only if

$$\begin{aligned} q^*(x_i)p(x_i)(1+r_u) &\geq q^*(x_i)p(x_i)(1+r_s) + 1 - q^*(x_i))p(x_i) \\ \Leftrightarrow\ q^*(x_i)p(x_i) &\geq \frac{1}{1+r_u-r_s} \end{aligned} \tag{9}$$

which holds with equality at signal $x_i = x^*_{s,u}$ when a financier is indifferent between both types of debt. Hence, using the expressions for $q^*(x_i)$, we obtain

$$q^*(x^*_{s,u})p(x^*_{s,u}) = \frac{1}{1+r_u-r_s} \quad \Leftrightarrow \quad F\left(\frac{x^*_{s,u}-\theta^*_s}{\sigma}\right)p(x^*_{s,u}) = \frac{1}{1+r_u-r_s} \tag{10}$$

Since $p(\cdot)$ and $F(\cdot)$ are strictly increasing, Eq. (10) admits a unique solution.

In deriving the thresholds x^*_s and $x^*_{s,u}$, we implicitly presumed the ordering $x^*_s \leq x^*_u$. In other words, we assumed the existence of a *catalytic effect on lending* so that by offering secured wholesale debt, the bank's likelihood of refinancing increases as secured creditors' critical threshold lies below the critical threshold of unsecured creditors in the benchmark model. The following Lemma provides a condition for this to hold.

Lemma 1. *Secured wholesale debt exerts a catalytic effect on lending if and only if*

$$x^*_s < x^*_u \quad \Leftrightarrow \quad r_s > \frac{r(1+r_u)}{1+r} \tag{11}$$

Proof. See Appendix. □

Lemma 1 suggests that secured debt need not necessarily lead to an improvement in funding availability. In particular, according to condition (11), if the interest rate on secured debt is too small compared to the risk free rate, the safety from holding collateral fails to compensate for the relatively higher interest gain of lending without collateral.

Using Lemma 1, the following Proposition fully characterizes the equilibrium of the funding model with three actions and two types of wholesale debt.

Proposition 2. *(Unique Refinancing Equilibrium in the Presence of Secured and Unsecured Debt)*

1. *If and only if condition* (11) *holds, there exists a unique threshold equilibrium summarized by* $\{x^*_s,\ x^*_{s,u},\ \theta^*_s\}$. *Financiers provide secured funding if and only if* $p(x_i) \geq p(x^*_s)$ *and they switch to providing unsecured funding if and only if* $p(x_i) \geq p(x^*_{s,u})$. *The bank obtains enough funding and survives until date* $t = 2$ *if and only if* $p(\theta) \geq p(\theta^*_s)$.

2. *Otherwise, whenever condition* (11) *fails to hold, the equilibrium features only unsecured debt and is summarized by the critical values* $\{x_u^*, \theta_u^*\}$.

3. *There are no other equilibria in non-threshold strategies.*

Proof. See Appendix. □

6. Equilibrium Properties

6.1. The Effect of Secured on Unsecured Lending

Lemma 1 provides the condition for secured debt to matter and to exert a catalytic effect on lending in equilibrium. It begs the question, though, how financiers' incentives to hold unsecured debt will be altered compared to the benchmark model. In fact, the availability of secured debt triggers two competing effects on financiers' incentives to provide unsecured debt. On the one hand, the catalytic effect exerts a *risk-reduction effect* as it mitigates the coordination problem and lowers the bank's funding risk. This makes financiers more willing to provide funding in general and *ceteris paribus* makes them also more willing to invest in unsecured debt. This leads to a crowding–in unsecured debt compared to the benchmark model. On the other hand, the reason behind the existence of the catalytic effect is that secured debt leads to a relatively higher expected payoff of secured debt compared to unsecured debt at signals $x_i \in (x_s^*, {}_{s,u}^*)$. *Ceteris paribus*, whenever the relative payoff advantage of secured debt becomes sufficiently strong, it may induce a crowding–out of unsecured debt compared to the benchmark equilibrium. Whenever this latter *payoff–effect* dominates, then $x_u^* < x_{s,u}^*$ and *vice versa* if the former risk–reduction effect is dominant. The following Proposition provides the respective conditions for crowding–in and crowding–out to occur.

Proposition 3. *(Crowding–Out and Crowding–In of Unsecured Debt)*

1. *If the coordination risk is sufficiently small,*

$$\frac{\gamma}{m} < \frac{r_s(1+r) - r(1+r_u)}{1+r_u} \in (0,1) \tag{12}$$

then $x_u^* < x_{s,u}^*$ *and the payoff–effect always dominates the risk–reduction effect. The availability of secured debt leads to a crowding–out of unsecured debt.*

2. *If condition* (12) *fails to hold and, in addition, the degree of balance sheet transparency is sufficiently large (σ sufficiently small), then $x_{s,u} < x_u^*$ and the risk-reduction effect dominates. The availability of secured debt leads to a crowding–in of unsecured debt.*

Proof. See Appendix. □

6.2. Efficiency of the Equilibrium Solution

It was shown above that unsecured creditors always refuse to provide funding to a bank that they believe to hold assets with negative NPV, i.e. $x_u^* > x_{\text{eff}}$. This insight continues to hold for unsecured financiers when secured debt is available.

Corollary 4. *(Efficient Funding Decision of Unsecured Financiers)*
Financiers provide unsecured debt only to banks that they believe hold assets with positive NPV.

Proof. See Appendix. □

However, a similar result does not exist for secured debt. In other words, financiers may provide secured debt also to banks holding negative NPV assets.

Corollary 5. *(Inefficient Funding Decision of Secured Financiers)*
Financiers provide secured debt also to banks that are believed to hold assets with negative NPV, i.e. $x_s^* < x_{eff}$, if and only if

$$\frac{\gamma}{m} < \Gamma \equiv \frac{(\ell - R_b)r_s - r(R_g - R_b)}{(\ell - R_b)r_s} \tag{13}$$

Proof. See Appendix. □

Also banks with negative NPV assets may be able to provide enough safety to secure financiers' initial investment by pledging collateral and thereby induce financiers to hold their debt. This occurs because essentially only the liquidation value of assets in case of default matter for making financiers' initial investment safe. Whether or not assets have positive NPV is irrelevant in this respect. When coordination risk falls *below* the bound provided by condition (13), then the bank provides too much collateral compared to the severity of the coordination problem that it faces and thereby it invites financiers to offer funding even if they believe that this is inefficient. However, inefficient funding decisions by financiers do not imply that banks with negative NPV assets necessarily survive. As in the benchmark model, this also depends on the degree of balance sheet transparency and on whether coordination risk is large or small, i.e. $\gamma/m \gtrless 1/2$.

Proposition 4. *(Continuation of Inefficient Banks)*

1. *Suppose* $\Gamma < \frac{\gamma}{m} < \frac{1}{2}$, *then financiers provide secured debt only to banks that they believe to be efficient. Nevertheless, there exists a critical degree of transparency* $\hat{\sigma}_s$, *such that inefficient banks obtain enough funding to continue if and only if their balance sheet transparency is sufficiently low, i.e.* $\sigma > \hat{\sigma}$.

2. *Suppose* $\frac{\gamma}{m} < \min\left\{\frac{1}{2}, \Gamma\right\}$, *then financiers provide secured debt even to banks that they believe to be inefficient and inefficient banks can continue independent of their degree of balance sheet transparency.*

3. *Suppose* $\frac{1}{2} < \frac{\gamma}{m} < \Gamma$, *then financiers provide secured debt even to banks that are believed to be inefficient. But inefficient banks obtain enough funding and continue if and only if their balance sheet is sufficiently transparent, i.e.* $\sigma < \hat{\sigma}_s$.

Proof. See Appendix. □

The previous results already shed some light on the welfare properties of secured debt. To simplify the analysis, we will restrict the following discussion to the limit when the degree of balance sheet transparency is infinitely precise, $\sigma \to 0$.[8]

[Figure 2 here]

Moreover, note that in the limit for $\sigma \to 0$, condition (13) becomes the necessary and sufficient condition for inefficient banks being able to refinance. Hence, if this condition fails to hold, then $\theta_s^* > \theta_{\text{eff}}$ and the introduction of secured debt leads to an unambiguous welfare gain.

If, however, condition (13) holds, then $p(\theta_s^*) < p(\theta_{\text{eff}}) < p(\theta_u^*)$. Denote by $\Delta(p(\theta))$ the difference in economic surplus between the case where secured debt is and where it is not available for a given $p(\theta)$,

$$\Delta(p(\theta)) = \begin{cases} 0, & \text{if } p(\theta) < p(\theta_s^*) \\ -(\ell - R_b) + p(\theta)(R_g - R_b) < 0, & \text{if } p(\theta_s^*) \leq p(\theta) \leq p(\theta_{\text{eff}}) \\ p(\theta)(R_g - R_b) - (\ell - R_b) > 0, & \text{if } p(\theta_{\text{eff}}) < p(\theta) < p(\theta_u^*) \\ 0, & \text{if } p(\theta) \geq p(\theta_u^*) \end{cases}$$

[8] In this case, $p(\theta_u^*) \approx p(x_u^*)$, $p(\theta_s^*) \approx p(x_s^*)$ and $p(\theta_{s,u}^*) \approx p(x_{s,u}^*) \approx p(\theta_{s,u}^*)$.

Note that $\Delta(p(\theta))$ is linearly increasing in $p(\theta)$ on $[p(\theta_s^*), p(\theta_u^*)]$ with $\Delta(p(\theta_{\text{eff}})) = 0$. Whether or not the introduction of secured debt leads to a welfare gain or a welfare loss depends crucially on the properties of $p(\theta)$ and on where in the interval $[p(\theta_s^*), p(\theta_u^*)]$ the efficiency threshold $p(\theta_{\text{eff}})$ is located. For example, whenever $p(\theta_{\text{eff}})$ is very close to $p(\theta_u^*)$ and relatively far away from $p(\theta_s^*)$, secured debt is likely to lead to an overall welfare loss.

Proposition 5. *(Welfare Gain Through Secured Debt)*
Given that secured debt exerts a catalytic effect on lending, i.e. condition (11) *holds, then for $\sigma \to 0$, the introduction of secured debt increases expected economic surplus if $p(\theta)$ is weakly concave in θ.*

7. Discussion and Conclusion

This paper provides a stylized global game banking model. The novelty of the paper is that we study a 3-action global game where financiers can invest into two secured and unsecured wholesale debt. First, we showed that the introduction of secured debt can exert a catalytic effect on lending. Second, depending the level of coordination risk and the degree of bank balance sheet transparency, secured debt can either lead to a crowding–out or a crowding–in of unsecured debt. Importantly, whenever coordination risk (and therefore bank funding risk) is large, crowding–in of unsecured debt may be observed. Third, secured financiers' care only about the liquidation value of the bank's assets. Whether or not the bank holds negative NPV assets does not matter to them and therefore the availability of secured debt may lead to inefficient continuation decisions of financiers. While this partial effect is detrimental to welfare, overall welfare tends to improve following the introduction of secured debt whenever the bank's terminal success probability is non-convex in the state θ.

While the model in this paper is stylized, it captures at least some aspects of the debate between those that claim that secured financing is currently "the only game in town", and therefore demand that policy makers should, at least temporarily, tolerate the shift to secured funding, and those that warn of the detrimental effects of this development.

Our model indeed suggests that, at least in the short term, secured funding may help financial institutions to overcome funding stress. It provides substance to the claim that secured funding may keep bank funding flowing under circumstances where it would otherwise come to a stop.

The model further suggests that during times when funding risk is rather high, secured funding can be issued without fearing that it will crowd–out unsecured financing for healthy banks. Nevertheless, the model indicates that in such cases, policy makers should be highly suspicious of banks that overly rely on secured funding. Banks with healthy balance sheets should be able to attract unsecured funding just as easily as secured funding. By contrast, an overly strong reliance on secured funding is likely to be associated with low quality assets.

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Appendix

Proof of Corollary 1. We first show that $x_u^* > x_{\text{eff}}$. The second claim then follows immediately.

1. If financiers provide funding, i.e. for any $x_i \geq x_u^*$,

$$(1+r)\ell \leq (1+r) \leq p(x_i)\left(1-\frac{\gamma}{m}\right)(1+r_u) < p(x_i)(1+r_u) < p(x_i)R_g < (1-p(x_i))R_b + p(x_i)R_g, \tag{A1}$$

   where the first inequality follows from Assumption 1 and the second one from Eq. (3). For any $x_i \geq x_u^*$, the inequality implies $p(x_i) > p(x_{\text{eff}}$.

2. Follows from the proof of (1) above.

□

Proof of Corollary 2. The proof follows immediately by differentiating θ_u^* with respect to σ and noting that the sign of the derivative changes if γ/m increases from below 1/2 to above 1/2. □

Proof of Corollary 3. The proof follows immediately by solving $p(\theta_u^*) = p(x^* + \sigma F^{-1}(\gamma/m)) < p(\theta_{\text{eff}})$ for σ, yielding

$$\hat{\sigma} \equiv \frac{x_u^* - \theta_{\text{eff}}}{-F^{-1}(\gamma/m)}$$

and by noting that the resulting inequality can hold for a sufficiently large σ if and only if $F^{-1}(\gamma/m) < 0$, which, by the symmetry of the distribution, is equivalent to $2\gamma < m$. □

Proof of Lemma 1. Since $p(\cdot)$ is strictly monotonic

$$x_s^* < x_u^* \quad \Leftrightarrow \quad p(x_s^*) < p(x_u^*)$$

Using Eqs. (5) and (7), the latter can be solved for r_s,

$$\frac{r}{r_s} < \frac{1+r}{1+r_u} \quad \Leftrightarrow \quad r_s > \frac{r(1+r_u)}{1+r}$$

which is the condition in the Lemma. □

Proof of Proposition 2. We first show that if and only if condition (11) holds, then $x_s^* < x_{s,u}^*$. Define the expected payoff difference between providing unsecured debt and secured debt conditional on the bank surviving for any $\theta \geq \theta_s^*$ as

$$\psi(x_i, x_s^*) \equiv F\left(\frac{x_i - x_s^*}{\sigma} - F^{-1}(\gamma/m)\right) p(x_i) - \frac{1}{1+r_u-r_s}$$

where θ_s^* has been replaced by its equilibrium value from Eq. (8). By Eq. (10), $\psi(x_i, x_s^*) = 0$ when evaluated at $x_i = x_{s,u}^*$. As $\psi(x_i, x_s^*)$ strictly increases in x_i, $\psi(x_s^*, x_s^*) < 0$ is necessary and sufficient for $x_s^* < x_{s,u}^*$,

$$\psi(x_s^*, x_s^*) < 0 \quad \Leftrightarrow \quad F\left(-F^{-1}(\gamma/m)\right) p(x_s^*) < \frac{1}{1+r_u-r_s}$$

Substituting $p(x_s^*)$ from Eq. (7) and using the symmetry of the distribution function allows to rewrite the last inequality as

$$r(1+r_u) < r_s(1+r)$$

which is condition (11). This implies that if and only if condition (11) holds, a monotone equilibrium can be constructed where agents switch from the lowest action (no funding) to the middle action (secured funding) at x_s^* and then switch from the middle action to the highest action (unsecured funding) at $x_{s,u}^*$. Since the 3-action funding game is a supermodular game and since the thresholds $\{x_s^*, x_{s,u}^*\}$ are unique, this equilibrium is unique by standard global game results for supermodular games.

Suppose that (11) fails to hold. We establish that the unique symmetric monotone equilibrium corresponds to the equilibrium in the benchmark game with only unsecured funding. Suppose that agents switch from no funding to unsecured funding at the threshold signal x_u^*, implying that the bank survives if and only if $\theta \geq \theta_u^*$. Note first that for any $x_i \geq x_u^*$, financiers always prefer to provide unsecured funding. To see this, write $\psi(x_i, x_u^*)$ as the payoff difference between unsecured and secured debt conditional on the bank surviving for any $\theta \geq \theta_u^*$. As $\psi(x_i, \cdot)$ strictly increases in x_i, a necessary and sufficient condition for unsecured debt to be preferred over secured debt at any $x_i \geq x_u^*$ is

$$\psi(x_u^*, x_u^*) > 0 \quad \Leftrightarrow \quad F\left(-F^{-1}(\gamma/m)\right)p(x_u^*) > \frac{1}{1+r_u-r_s}$$

Substituting $p(x_u^*)$ from Eq. (5), the last inequality becomes

$$\frac{1+r}{1+r_u} > \frac{1}{1+r_u-r_s} \quad \Leftrightarrow \quad r(1+r_u) > r_s(1+r)$$

which is equivalent to the assumption that Eq. (11) fails to hold. To complete the proof, note that from the benchmark model, financiers never switch back to refusing funding for any $x_i \geq x_u^*$. Thus, whenever Eq. (11) fails to hold, financiers in equilibrium switch from no lending to unsecured lending at x_u^* and never switch back to one of the two lower actions "secured lending" or "refusing funding".

Proof of Corollary 4. If financiers provide unsecured funding, i.e. for any $x_i \geq x_{s,u}^* \geq x_s^*$,

$$(1+r)\ell \leq (1+r) \leq \frac{1}{F((x_{s,u}^*-\theta_s^*)/\sigma)} + r = p(x_{s,u}^*)(1+r_u-r_s) + r = p(x_{s,u}^*(1+r_u) + (p(x_s^*)(1-\gamma/m) - p(x_{s,u}^*))r_s$$

$$< p(x_{s,u}^*)(1+r_u) + (p(x_s^*) - p(x_{s,u}^*))r_s < p(x_{s,u}^*)(1+r_s) < p(x_{s,u}^*)R_g < p(x_i)R_g < (1-p(x_i))R_b + p(x_i)R_g$$

endproof

□

		Other creditors		
		do not lend	lend secured	lend unsecured
Creditor i	do not lend	$1+r$	$1+r$	$1+r$
	lend secured	1	$p(\theta)(1+r_s)+(1-p(\theta))$	$p(\theta)(1+r_s)+(1-p(\theta))$
	lend unsecured	0	$p(\theta)(1+r_u)$	$p(\theta)(1+r_u)$

Table 1: Payoffs in extended game with secured and unsecured wholesale debt.

Figure 1: The global game solution for $\sigma \to 0$ in the benchmark model with unsecured debt.

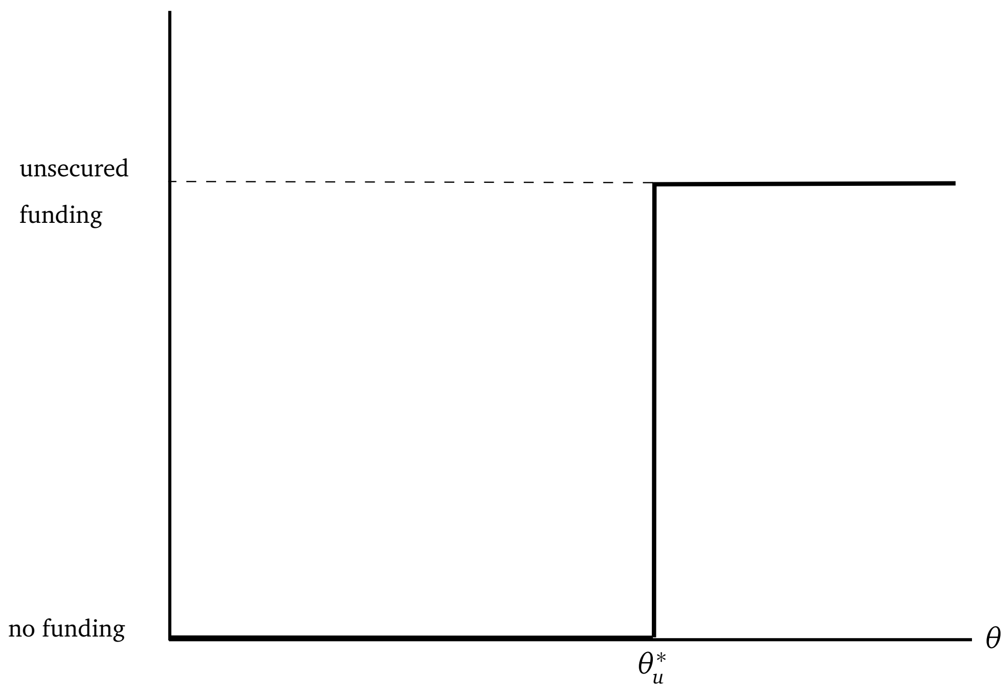

Figure 2: The global game solution for $\sigma \to 0$ in the model with secured and unsecured debt.